THEORY OF MACHINE

HANDWRITTEN NOTE WITH NUMERICALS

FOR 3RD YEAR 1ST PART

BY

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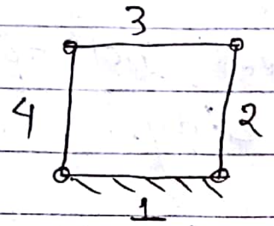
BAG 072

- Arjun Gautam

Simple Mechanism

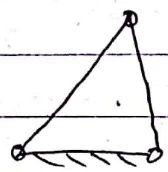
Mechanism :-

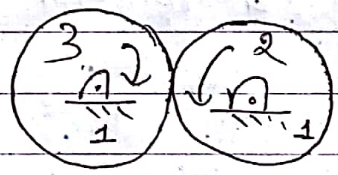
When one link of a kinematic chain is fixed, then it is called mechanism.



No. of link = 4.
If one fixed, it is mechanism

- For become a mechanism, there must be four links. Three may lock. But changing shape can make 3 links to a mechanism


locked



it is mechanism.

→ Mechanism is only that when we provide an input then ar predictable output will come.

Kinematic link or element :-

Kinematics → branch of mechanics that concerned with the motion of object without refrence to the forces that cause the motion.

④ Kinematic link or element :-
Each part of machine which moves relative to each other is called kinematic link.

- kinematic link or element need not to be rigid body but it must be resistant body.

rigid body = no deformation. e.g: we can assume piston & piston rod

resistant ~~resultant~~ body = slightly deformation e.g:- belt moving Shaft & flywheel

Characteristics of kinematic link :-
(i). must have relative motion
(ii) must be a resistant body.

Types:
① Rigid link :- theory only, do not deform, e.g:- connecting rod, crank deformation (assumed).
② Flexible link :- partly deformed, belt, ropes etc.
③ Fluid link :- motion transmitted through fluid by pressure or compressor e.g: hydraulic brake

④. How to find kinematic chain, pair & all ?
⇒ Answer → turn few pages.

(7). Kinematic pair :-
Two links or elements of a machine, when in contact with each other, are said to be formed a pair. There must be a relative motion betⁿ the elements.

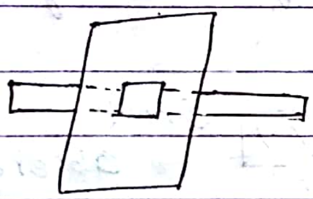
Motion may be completely, incompletely or successfully constrained.

It is very imp. to know their definition & differences

(a). completely constrained motion:-
when motion betⁿ a pair is limited to a definite direction irrespective to direction of force applied.

input ⟶ single predictable output

e.g:-

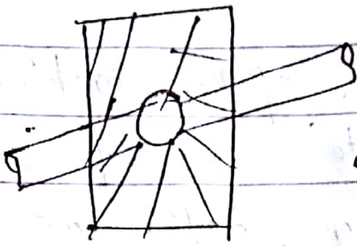


Square hole with square rod.
input one → output one i.e. reciprocating or sliding motion

(b). incompletely constrained motion:-
→ motion betⁿ a pair can take place in more than one direction.

single Input ⟶ more output
(sliding & rotating)

e.g:-

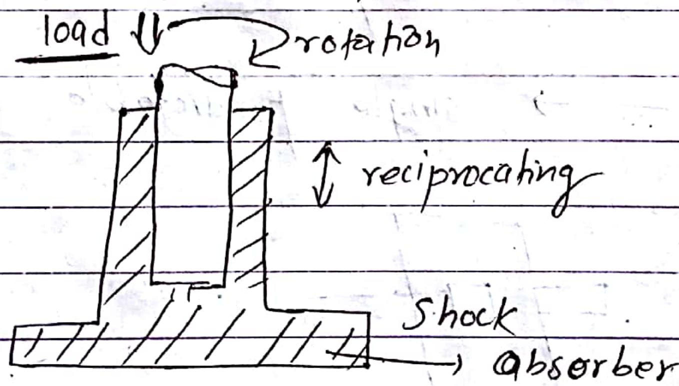


output → rotating & sliding

②. Successfully constrained motion:-
motion that is not fixed sometime complete sometime incomplete.

Defn:-
motion betn the elements, forming a pair is such that the constrained motion is not completed by itself, but by some other means, then this motion is successfully constrained motion.

e.g:



when load is not added, two motion occured
(i). rotatory
(ii) reciprocating so, it is incompletely constrained

But when load is added only one ~~reciprocating~~ rotatory motion occured. it is completely constrained
This is altogether is called successfully constrained. (कहिले completely कहिले incompletely बनाउन मिल्ने)

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Classification of kinematic pair:

(i). on the basis of relative motion betn the elements
(ii) on the basis of type of contact.

(1). On the basis of relative motion betn the elements:

(a). Sliding pair: DOF = 1

(b). Turning pair or hinged or revolute pair: DOF = 1

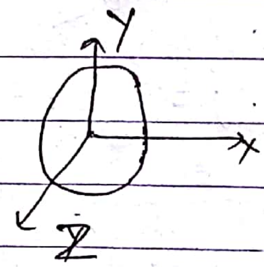
(c). Rolling pair: one roll over another fixed link, DOF = 2

(d). Cylindrical pair: DOF = 2

(e). Spherical pair: DOF = 3

(f). Screw pair: DOF = 2

why 3 ?

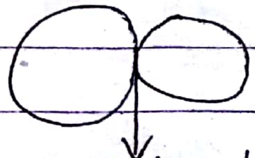


Here, 3 axis (x, y, z) a sphere can move in all x, y, z dirn, but Moment of inertial along x-axis, y-axis & z-axis is zero. So, no rotation occur that's why DOF = 3

(2). On the basis of type of contact

(a). Lower pair → surface contact
e.g.: hinge pair

(b). Higher pair → point or line contact, roller pair



higher pair

(4) Kinematic Chain :-

When the kinematics links are connected by no. of pair such that they coupled with last link to first or first to last link to transmit definite motion i.e. completely or successfully constrained. Then it is called kinematic chain.

OR

It may be defined as a combn of kinematic pairs, joined in such a way that each link form a part of two pairs and the relative motion betn links or element is completely or successfully constrained.

To be a kinematic chain

$\boxed{l = 2p - 4}$ where, l = No. of link
p = No. of pair

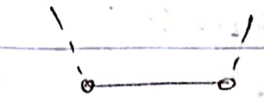
OR

$\boxed{j = \frac{3}{2}l - 2}$ where j = No. of joint
l = No. of pair

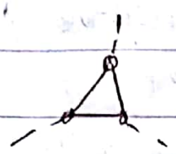
How to prove in few page later.

Types of link & joints.

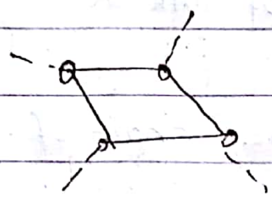
1. Binary link



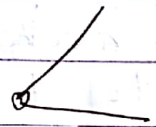
2. Ternary link



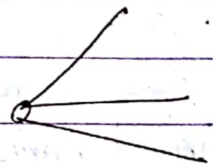
3. Quarternary link



1. Binary joint

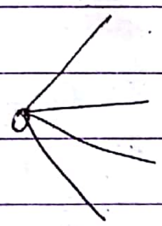


2. Ternary joint



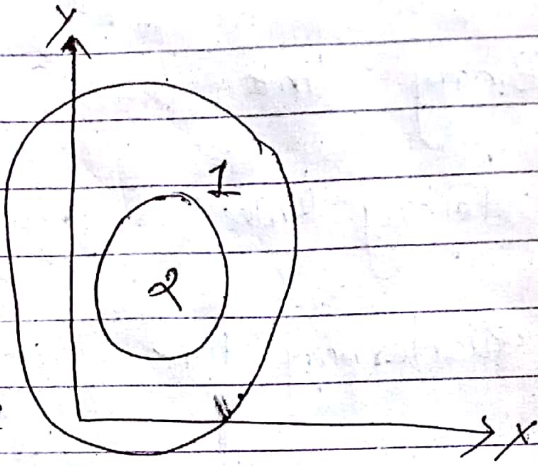
1 ternary joint = 2 binary joint

3. Quartornary joint



1 quarternary joint = 3 binary joint

Gruebler's criterion for constraint motion for planar mechanism with lower pair :-



Consider link 2 has a relative motion w.r.t. link 1. The max^m Degree of freedom (DOF) of link 2 $f_{max} = 3$.

Consider a kinematic chain of "n" link connected together by p no. of lower pair. If the kinematic chain form a mechanism, one link of kinematic chain is fixed. Let F be the degree of freedom of the considered mechanism. Then,

$$F = 3(n-1)$$

For p no. of lower pair in kinematic chain, the DOF of the mechanism is $F = 3(n-1) - 2p$

$$F = 3(n-1) - 2p$$

if $F = 1$

$$1 = 3(n-1) - 2p$$

$$\boxed{3n - 2p = 4} \quad \text{———} \quad (1)$$

The eq^n (1) is called gruebler's eq^n.

If kinematic chain are made up of different types of link. Then,

let, n_2 = no. of binary link
n_3 = no. of ternary "
n_4 = no. of quarternary "

Then,

$$n = n_2 + n_3 + n_4 + n_5 + n_6 \cdots n_i$$
$$2p = 2n_2 + 3n_3 + 4n_4 + 5n_5 + \cdots in_i \quad \text{(II)}$$

from eqn (I) & (II)

$$F = 3(n_2 + n_3 + n_4 + n_5 + \cdots - 1) - (2n_2 + 3n_3 + 4n_4 + 5n_5 + \cdots)$$

$$= (3n_2 + 3n_3 + 3n_4 + 3n_5 + \cdots - 3) - (2n_2 + 3n_3 + 4n_4 + 5n_5 \cdots)$$

$$F = n_2 - n_4 - 2n_5 - 3n_6 - 3$$

$$\boxed{F + 3 = n_2 - (n_4 + 2n_5 + 3n_6 + \cdots)} \quad \text{(III)}$$

$$n_2 = (F+3) + (n_4 + 2n_5 + 3n_6 + \cdots)$$

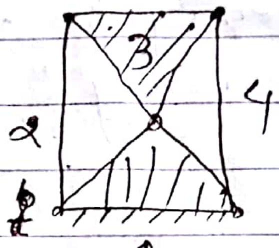
If $F = 1$, $n_2 \geq 4$ [binary joints must be greater than 4 or 5]
$F = 2$, $n_2 \geq 5$ [resp.

Higher order of link = $\frac{\text{no. of link}}{2}$

for 4 bar mechanism,
higher order of link = $4/2 = 2$ (is binary link)

for 6 bar mechanism,
higher order of link = $6/2 = 3$ (is Ternary link)

For binary link to find



No. of link = 4

No. of joint = ?

Binary joint = 4

Ternary joint : 2

How to find from eqn (11)

If $F=1$

$$4 = n_2 - (n_4 + 2n_5 + 3n_6 + \cdots)$$

$$4 = n_2 - 0 \quad \text{[Since, it doesn't exceed } n_3]$$

So, $n_2 = 4$ [minimum binary joints]

for eight bar mechanism

higher order of link $= \frac{8}{2} = 4$

For $f=1$

$$n_2 - n_4 = 4$$

$$n_2 + n_3 + n_4 = 8$$

So, the no. of binary, ternary or quartenary joints can be found in such a way e.g.:

n_2	n_3	n_4
4	4	0
5	2	1
6.	0	2

Degree of Freedom (DOF)

DOF is defined as the number of input parameters (usually pair variables) which must be independently controlled in order to bring the mechanism into a useful engineering propose.

formula to find degree of freedom:-

$$n = 3(l-1) - 2j - h$$

where,

n = no. of DOF

l = no. of link

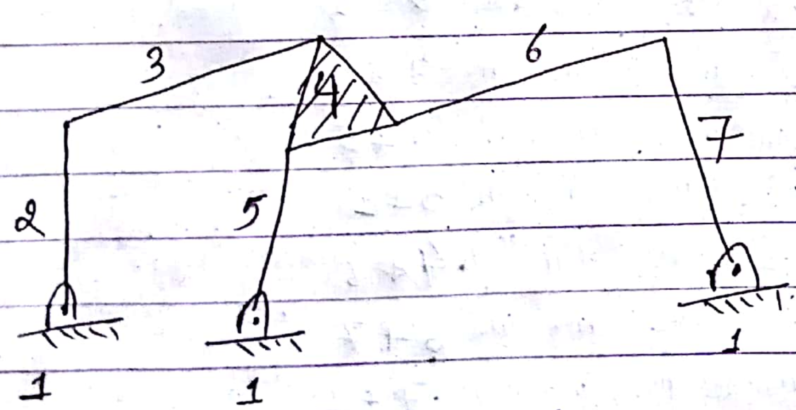
j = no. of joint ~~pair~~ (binary ~~only~~)

h = no. of higher pair

Steps:- to find DOF, link, pair

1. Finding kinematic link :-

If relative motion changes then no. them to new link. for fixed, link no. provided is same. e.g:-



∴ No. of link = 7

(a). Give no. 1 to all fixed link 1st.

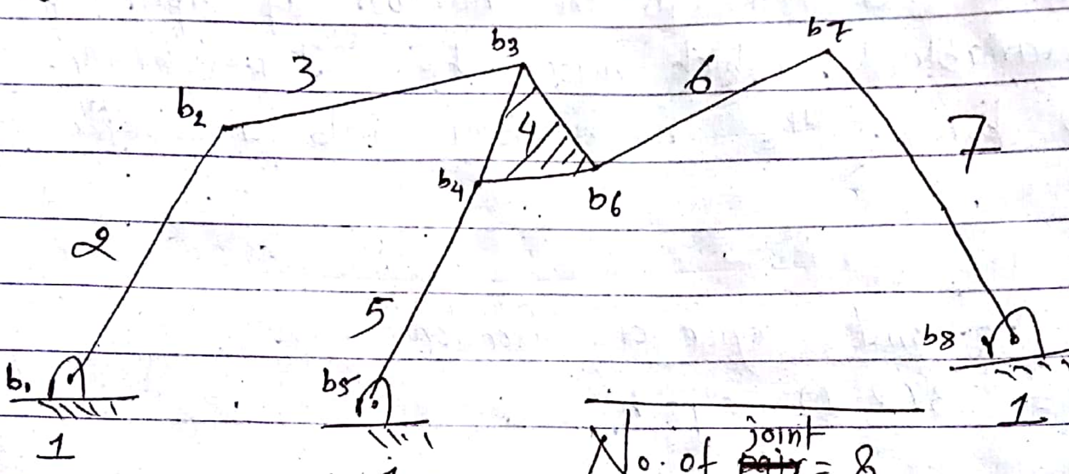
(b) Since, looking to left, at hinge point the motion changes so, name (2) for the link connecting to hinge

(c). Ily, changes motion name 3

(d). One ternary link, so name 4

(e) Go same process

② finding ~~kinematic pair~~ binary joint



No. of ~~pair~~ joint = 8

Fig-①

Steps:

ⓐ. Here, link 1 & 2 form a binary ~~pair~~ joint.
(યદિ link no. change છે તો ~~pair~~ joint બદલાય છે ~~હતી~~.
1, 2 link ના વચ્ચે binary ~~pair~~ joint છે ।)

ⓑ. Name binary ~~pair~~ joint to b which makes easy to find nos. of binary ~~pair~~ joint as above.

b_1 = 1st binary ~~pair~~ joint of link 1 & 2
b_2 = 2nd " " " " 2 & 3
b_3 = 3rd " " " " 3 & 4
b_4 = 4th " " " " 4 & 5
b_5 = 5th " " " " 5 & 1
b_6 = 6th " " " " 4 & 6
b_7 = 7th " " " " 6 & 7
b_8 = 8th " " " " 7 & 1

∴ No. of ~~pair~~ joint = 8

Note

If ternary joint come then,

1 ternary joint = 2 binary joint

lly, 1 quartenary " = 3 " ".

(2) finding higher pair.

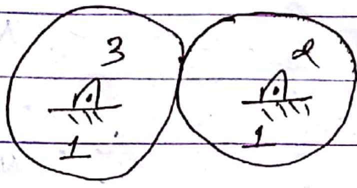


fig - (11)

If 1 pt. or line contact is there, then we obtain 1 higher pair.

So, $h = 1$

$l = 3$

$j = 2$

To prove if it is kinematic chain or not.

fig (1).

No. of kinematic link = 7 = l

" " " pair = 8 = p

" " higher " = 0 = h

So, " " binary joint = 8 = j

Formula: $l = 2p - 4$

or, $7 = 2 \times 8 - 4$

or, $7 = 16 - 4$

$7 = 12$

L.H.S < R.H.S

$j = \frac{3}{2} l - 2$

or, $8 = \frac{3}{2} \times 7 - 2$

or, $8 = 10.5 - 2$

or, $8 = 8.5$

$8 < 8.5$

So, no, kinematic chain LHS < RHS
unconstrained chain

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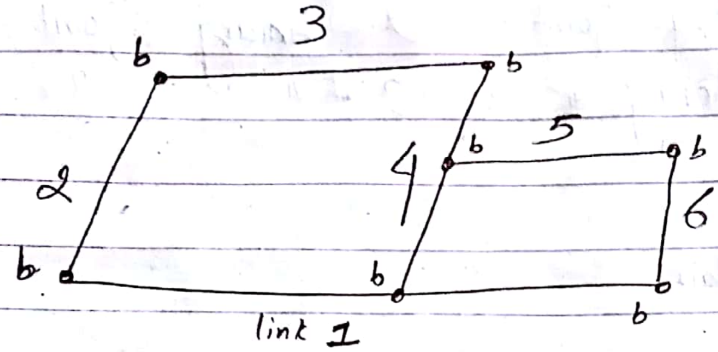
Fig (a)

(2)



Note :- No. change motion so, no link change

No. of link (l) = 6

No. of pair (p) = 5

Then, No. of joint (j) = ~~6~~ 7.

$l = 2p - 4$

$6 = 2 \times 5 - 4$

$6 = 10 - 4$

$6 = 6$

$j = \frac{3}{2} l - 2$

$7\cancel{6} = \frac{3}{2} \times 6 - 2$.

$7\cancel{6} = 9 - 2$

$7 = 7$

Note :-

1, 2 → 1 pair

2, 3 → 1 pair

3, 4 → 1 pair

5, 6 ~~4, 5~~ → 1 pair

6, 1 → 1 pair

~~6, 1 → 1 pair~~

5 pair

∴ LHS = R.H.S (contrained kinematic chain)

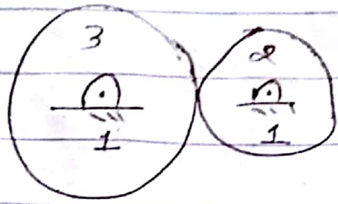
<u>Summary</u>

LHS = RHS = compound chain (more than 4 pair), contrained (4 pa

LHS < RHS = unconstrained chain

LHS > RHS = locked chain

(3)



No. of link = 3

No. of higher pair = 1

No. of joint = 2

So, $j + \frac{h}{2} = \frac{3}{2} l - 1$

$2 + \frac{1}{2} = \frac{3}{2} \times 3 - 1$

or, $\frac{5}{2} = \frac{9}{2} - 1$

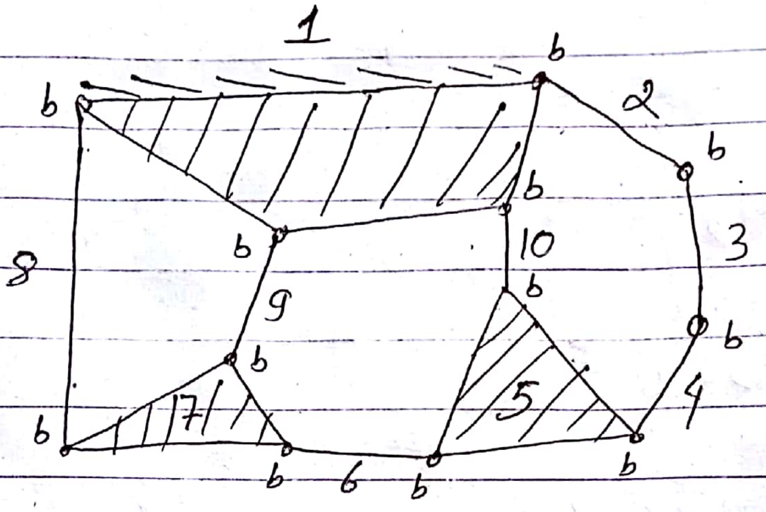
or, $\frac{5}{2} = \frac{5}{2}$

constrained kinematic chain (LHS = RHS)

Finding DOF

Questions: for practice

(1)



No. of link $(l) = 10$

No. of binary joint $(j) = 12$

No. of higher pair $(h) = 0$

$$DOF = 3(l-1) - 2j - h$$

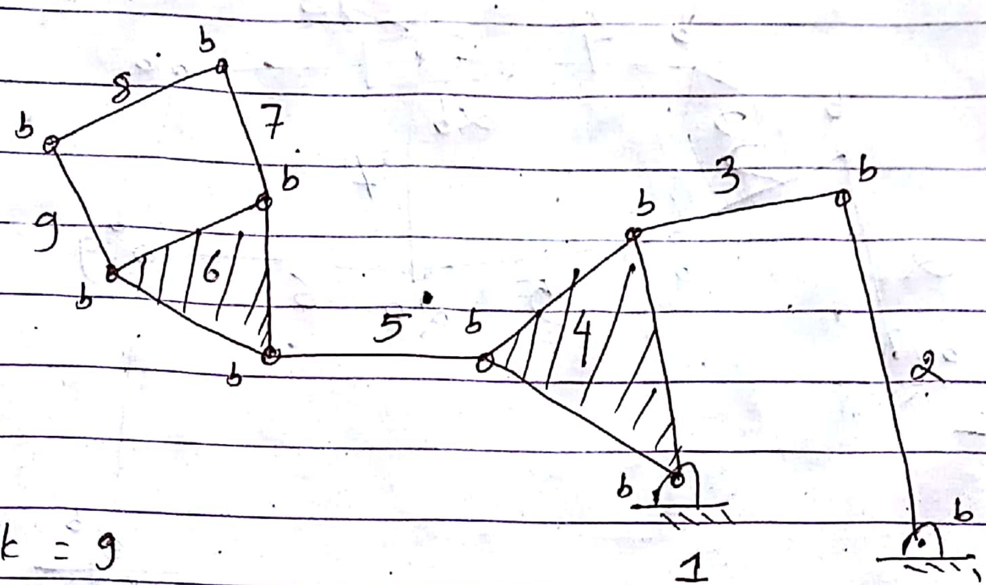
$$= 3(10-1) - 2 \times 12 - 0$$

$$= 3 \times 9 - 24$$

$$= 27 - 24$$

$$\boxed{DOF = 3}$$

②.



No. of link = 9

No. of joint = 10

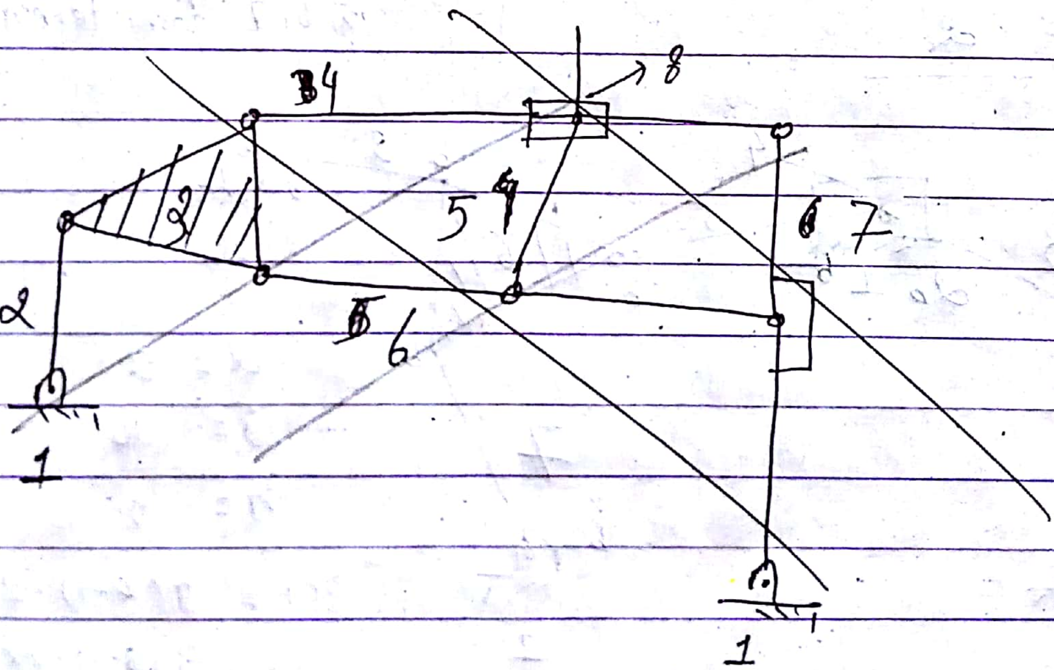
No. of higher pair = 0

$$DOF = 3(l-1) - 2j - h$$

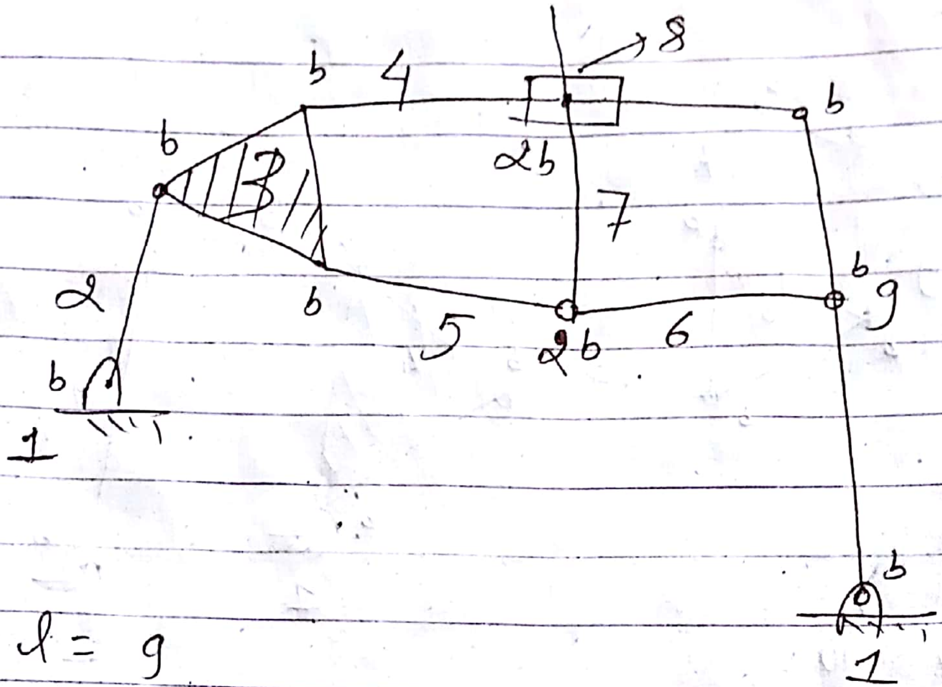
$$= 3(9-1) - 2\times10 = 3\times8 - 20$$

$$\boxed{DOF = 4}$$

③.



③



$l = 9$

$j = 11$

$h = 0$

$$DOF = 3(l-1) - 2j - h$$

$$= 3(9-1) - 2 \times 11 - 0$$

$$= 3 \times 8 - 22$$

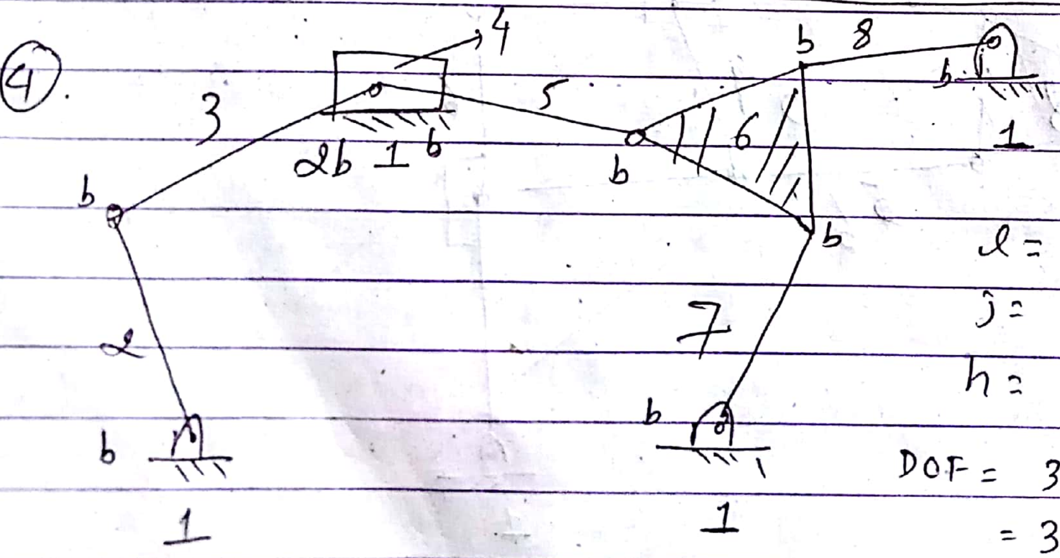
$$DOF = 2$$

Note:
In slider, 4, 7 & 8 are ternary joint, so, binary joint = 2b

Also,

5, 6, 7 form ternary

④.



$l = 8$

$j = 10$

$h = 0$

$$DOF = 3(l-1) - 2j - h$$

$$= 3(8-1) - 2 \times 10 - 0$$

$$= 21 - 20$$

$$DOF = 1$$

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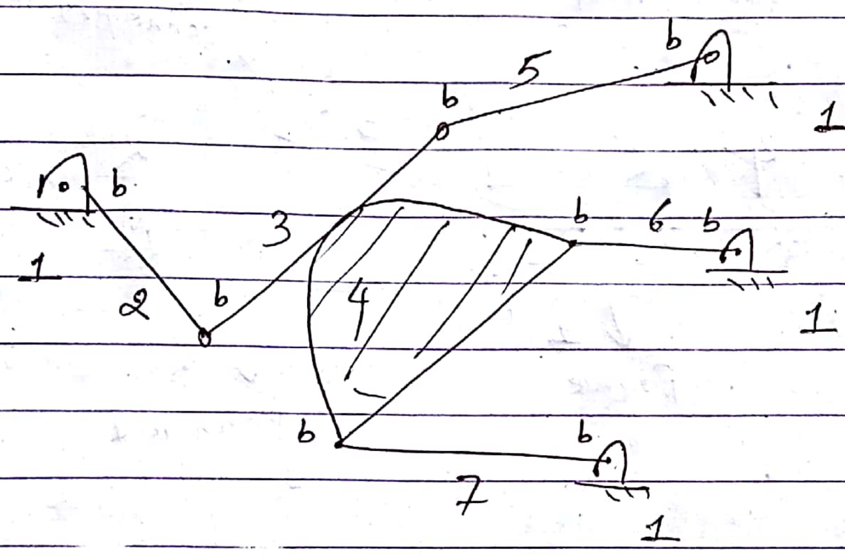
Note

look at 4^{th} link

1, 4 make one binary joint

3, 4 & 5 are connected make 1 ternary joint equals to 2 binary joint



$l = 7$

$j = 8$

$h = 1$ [in 3 & 4 link]

$$DOF = 3(l-1) - 2j - h$$

$$= 3(\cancel{8}7-1) - 2 \times 8 - 1$$

$$= 3 \times 6 - 16 - 1$$

$$= 18 - 16 - 1$$

$$DOF = \underline{1}$$

⊛. Inversion of Mechanism :-

The process of fixing different link of same kinematic chain to obtain different mechanism is called inversion of mechanism.

Single Slider mechanism

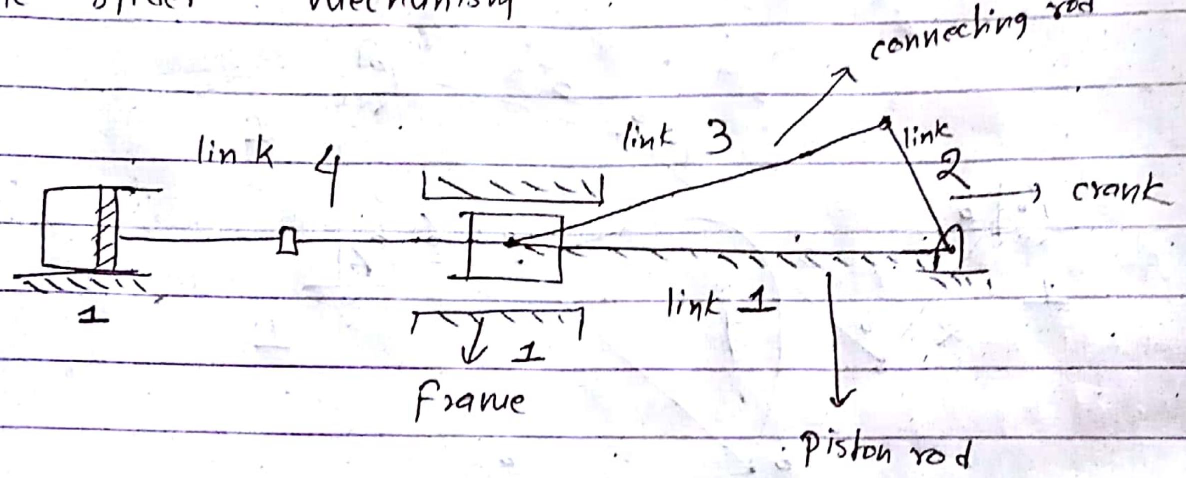


fig:- Single slider mechanism

Inversion of Single Slider Mechanism :-

②. Since, there are 4 links in slider so, we can obtained 4 different mechanism by fixing 4 diff. link as follows:

①. Oscillating cylinder engine

⊛ when connecting rod or link 3 is fixed (connecting rod)

⊛. It is used to convert reciprocating motion into rotatory motion.

②.

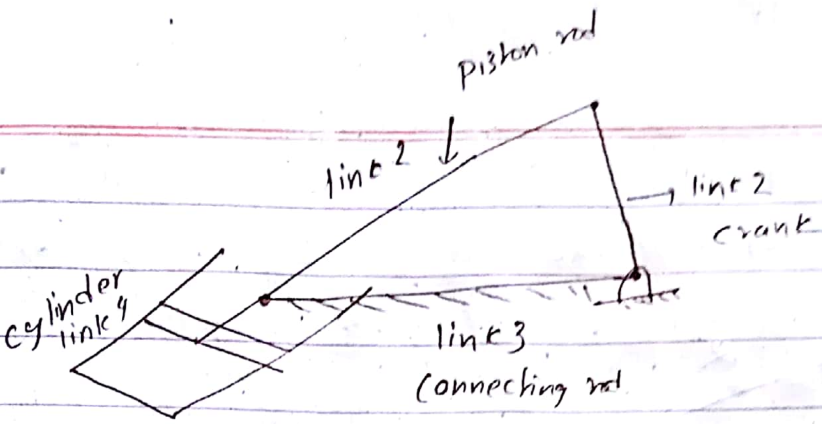
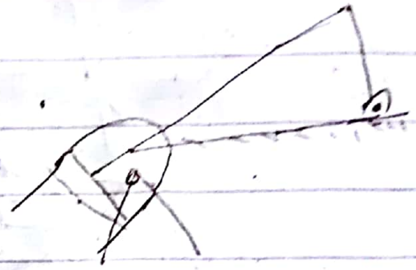


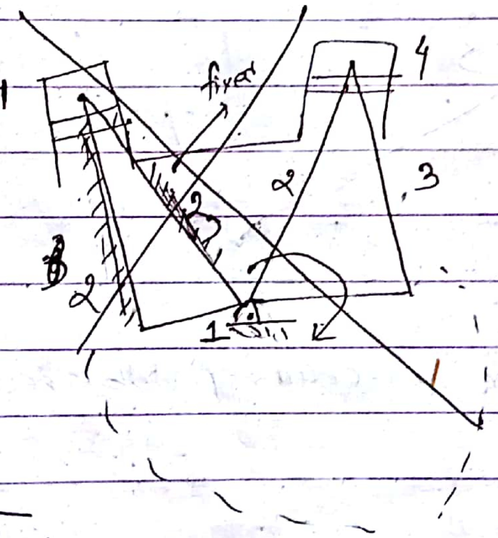
Fig-①. when connecting rod is fixed

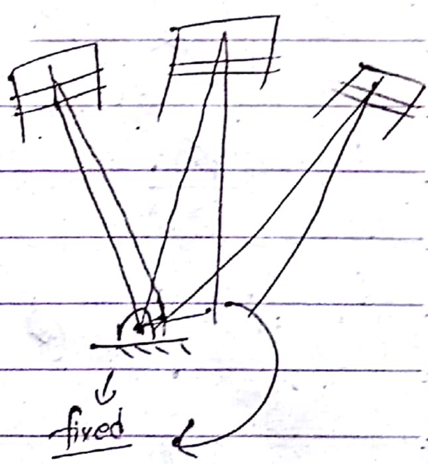


②. Rotatory internal combustion engine or Gnome Engine:-

*. If ~~piston rod~~ crank is fixed we obtain such mechanism

*. when connecting rod (link 3) rotates, the piston link (4) reciprocates inside the cylinder

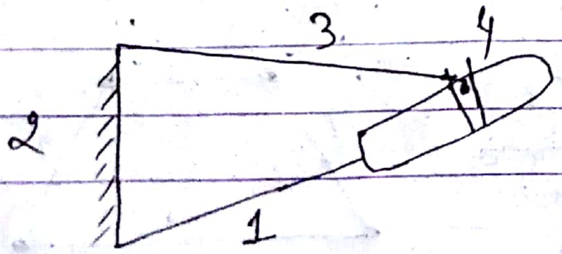




Not important

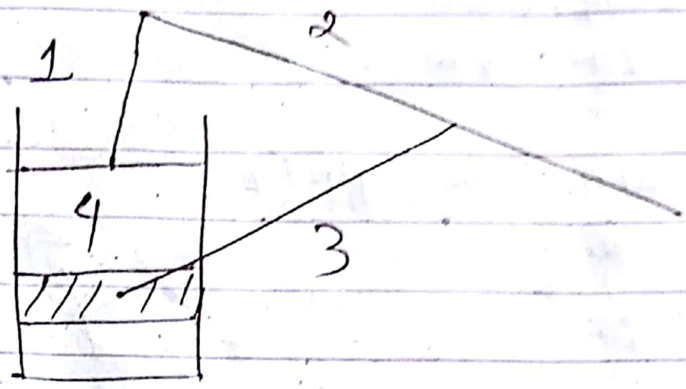
③. If 2nd link Crank is fixed, ① rotary engine

② whiteworth quick return mechanism

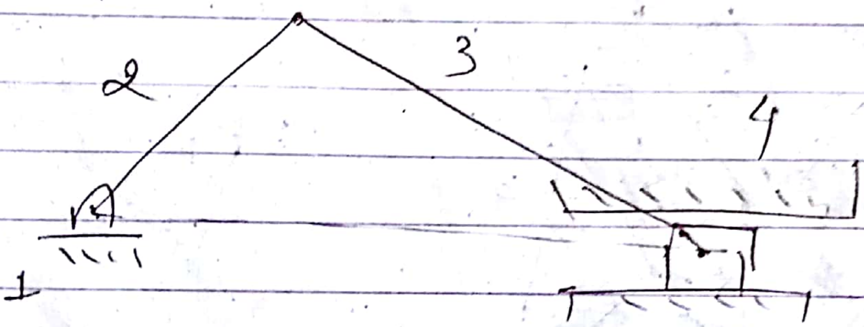


③. Hand pump

(*) If link 4 is fixed



④. If 1st crank is fixed



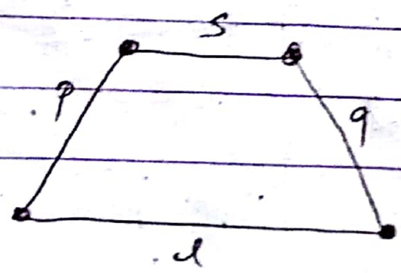
∴ (*) Slider mechanism obtained

Greshof's law:-

It states that " If 'l' & 's' be the length of longest and smallest length of linkage and 'p' & 'q' be the length of the remaining other two elements of the planar four bar mechanism then

$l+s \leq p+q$ "

From Greshof's law, we get continuos motion



(*). If the mechanism satisfies above equation, it is called class I mechanism.

(*). If not it is called Class (II) or Rocker-Rocker mechanism.

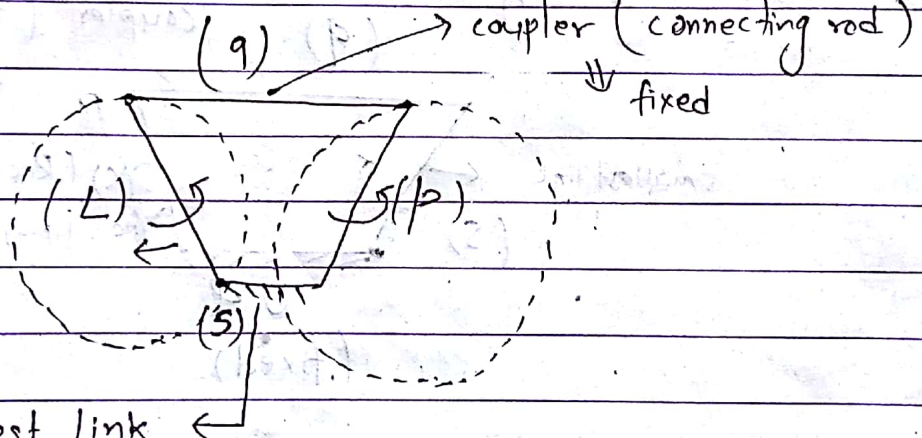
Rocker - Rocker mechanism ⇒ If input is oscillating then output is also oscillating.

smallest link = crank

Inversions :- $\boxed{(S+L<P+Q)}$

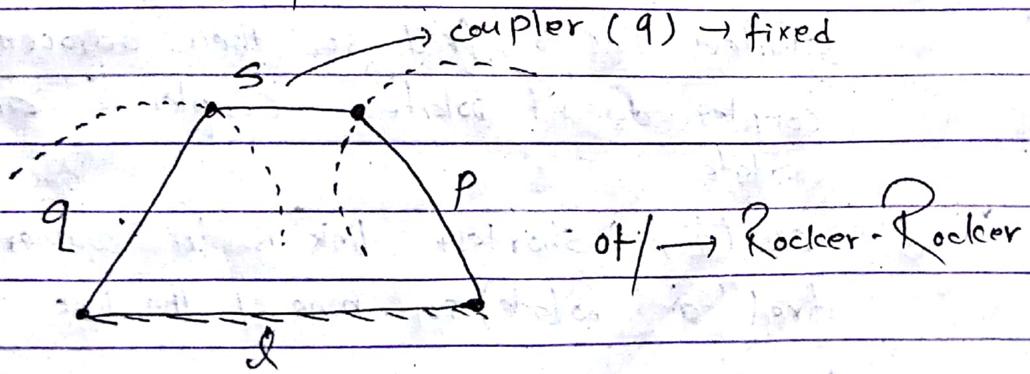
case

1) If smallest link (crank) is fixed, then crank-crank mechanism is obtained



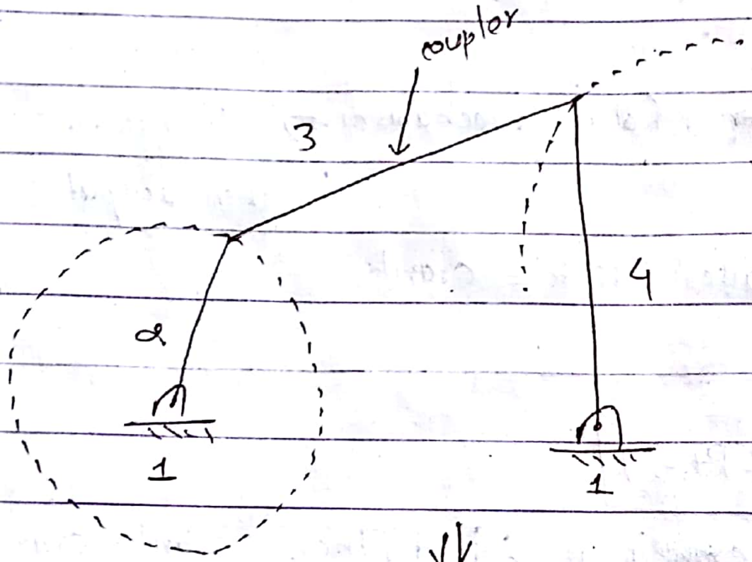
~~crank longest link is~~

case 2:- If smallest link is coupled, Rocker-Rocker mechanism is obtained



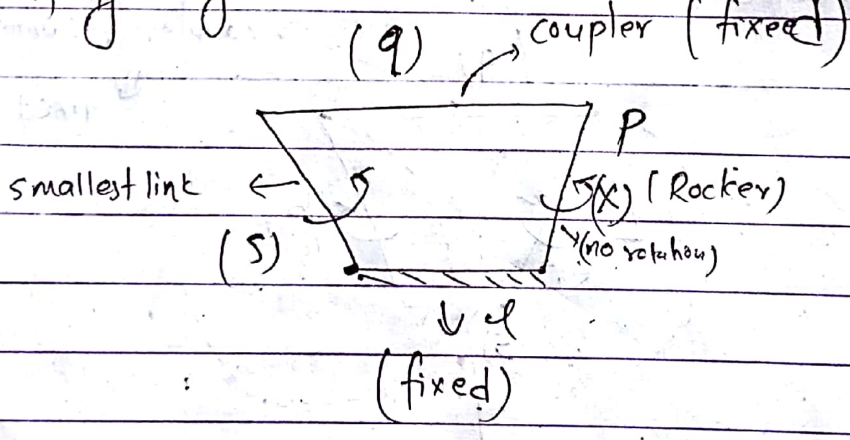
Case III.

If smallest link is crank, any one of the adjacent link is fixed, <u>Crank- Rocker mechanism</u> is formed.



Application field:
① mini-drafter

Simplify way



<u>How to remember</u> :-

①. Smallest link always wants to rotate so, in case I smallest link is fixed so, their adjacent link rotate but coupler doesn't rotate. Coupler is always fixed it cannot rotate.

②. case (II) Shortest link is in coupler position, ie. coupler is fixed (can't rotate) so, none of the link rotate. Its adjacent

(4). link can't be rotate, motion can't be transfered here.

(4). Case (III), when smallest link is adjacent to any fixed link then, smallest link rotate but coupler remain fixed as well as due to not transfering motion another link(p) also can't rotate.

Analysis of Mechanism:

It is used to determine the motion characteristics of mechanism of known geometry (length, width, angle given). The velocity & acceleration of the diff. ~~position~~ points on the element of mechanism are determined.

methods:-

- → Relative method
- → Instantaneous centre method (only applicable for velocity)

Synthesis of mechanism:

It is used to design the mechanism for specified motion characteristics.

Relative method:-

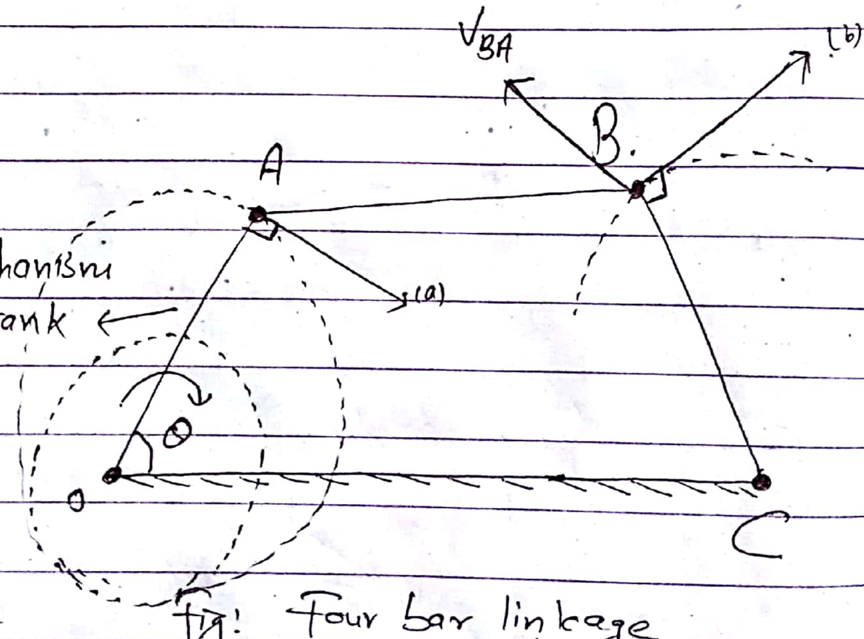


Fig: Four bar linkage

Consider a 4-bar mechanism in which crank OA rotates with uniform angular velocity (ω) in clockwise dir^n.

For given configuration of the mechanism, velocity at point A w.r.t pt. O is

$V_A = \omega \cdot OA$ (absolute vel.)

NOTE:

angular vel हुँदा angular accl^n zero हुन्छ fixed को Respect मा → absolute acceln हुन्छ।

- Arjun Gautam

How to draw vector diagram:

Steps:- (1). Take fixed point or refrence point

(2) Draw oa ⊥ OA to represent the mag. & dir. of V_n. (watch vector diagram)

(3). Draw ob ⊥rd BC to represent $\vec{V_B}$.

(4). Draw a line through pt. 'a' ⊥rd to AB to intersect a pt. b on OB

NOTE:

we know only a mag. & dir. of (a), So, vel. of B w.r.t. a B possible but vel. of A w.r.t. B is not possible.

Here,

$V_B = ob \times$ scalar factor

$= (Ans.)\ m/s$

$V_{BA} = ab \times$ scalar fact

$= (Ans.)\ m/s$

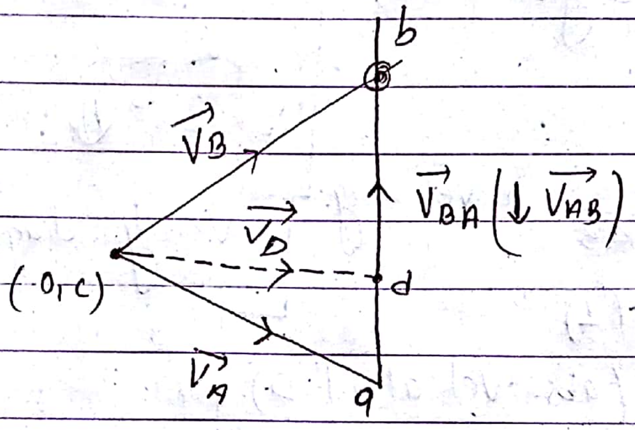


Fig: velocity vector diagram - fig (1)

Now,

from velocity vector diagram

$V_{AB} = \omega_{AB} \cdot AB$ $\quad (V = \omega r)$

$ab = \omega_{AB} \cdot AB$

$\omega_{AB} = \dfrac{ab \rightarrow \text{velocity}}{AB \rightarrow \text{magnitude}}$

$= (Ans.)\ rad/sec$ (5)

For link BC

$$\omega_{BC} = \frac{V_B}{BC}$$

$$= \frac{ob}{BC} = (\text{Ans.})\ \text{rad/sec}$$

For intermediate point on link AB

$V_D = ?$

Given;

$$\frac{AD}{AB} = \text{given (ratio)}$$

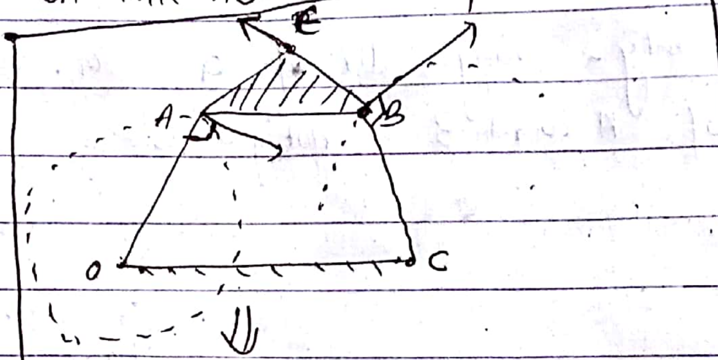
Take,

$$\frac{ad}{ab} = \frac{AD}{AB} \text{ in vel. diag.}$$

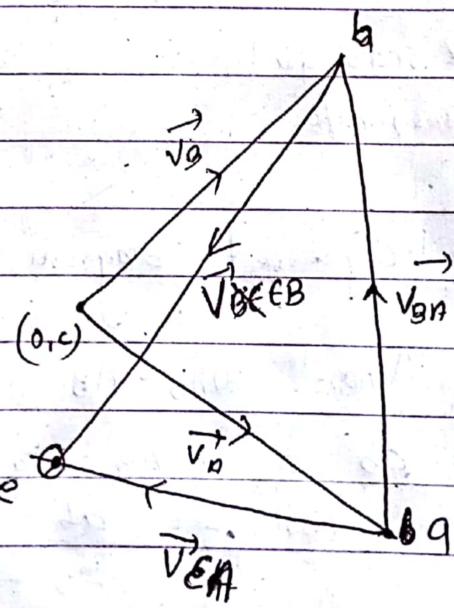
from above fig (1)

$V_D = od$ (abs. vel at pt. D)

At point E



Velocity diagram:- Fig: 2



ω

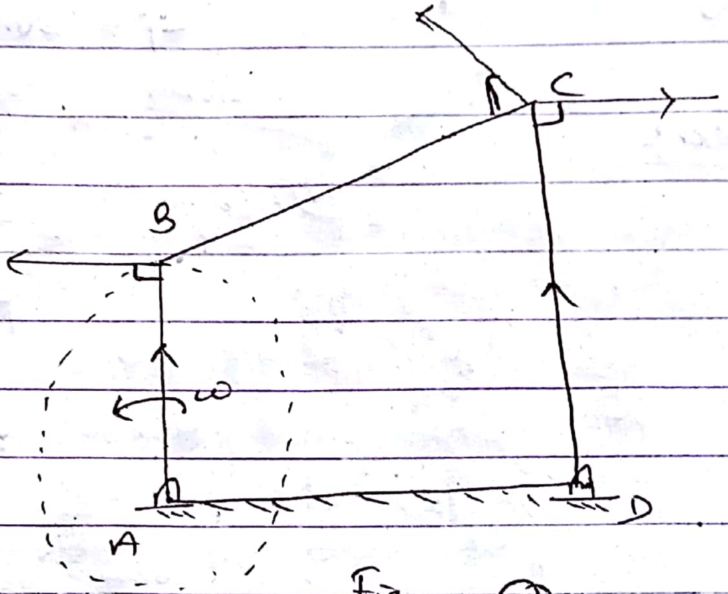
Numerical Questions:-

Q. No. 1

Note :-

When two links of 4 bar are parallel to each other both velocity vector coincides. If one is known other can be obtained.

e.g:-



Here from fig ⓐ
AB || CD so,
velocity diagram will be

Process:

1. Draw refrence point (a,d)
2. Draw ab ⊥ AB
3. From point B, Draw BC ⊥ passing through BC
4. Draw CD ⊥ ac, it fall on line ab (Since they are parallel)

Fig — ⓐ

Now,

$\vec{V_B} = W_{AB} \cdot AB$

$W_{AB} = \dfrac{\vec{V_B}}{AB}$ (W_{AB} = given)

$W_{AB} = \dfrac{ab}{AB}$

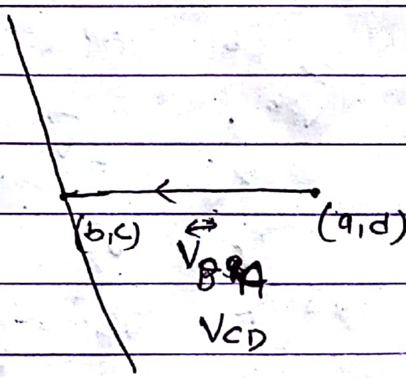


Fig: Vector diagram

From vel. diagram

$V_{BA} = V_{CD}$ (or, $V_B = V_C$)

$W_{AB} \cdot AB = W_{CD} \cdot CD$

$$\boxed{W_{CD} = \frac{W_{AB} \cdot AB}{CD}}$$

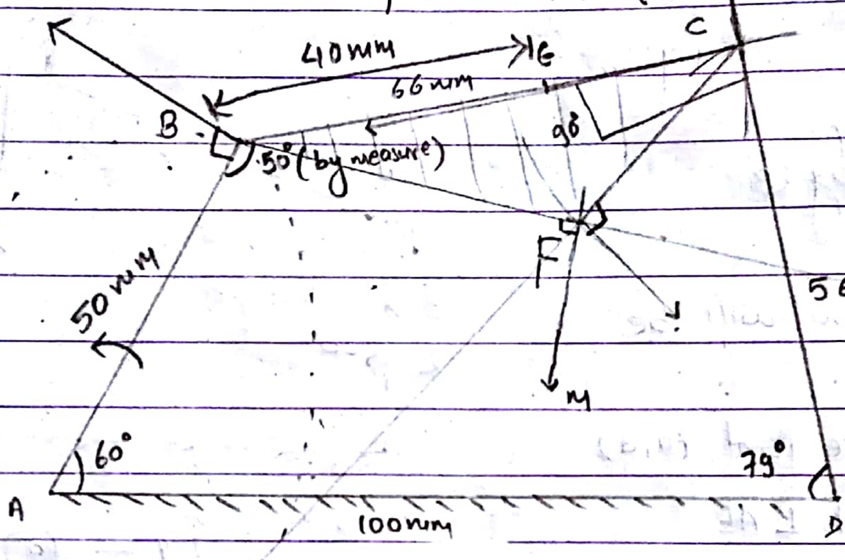
Easily found out

$\boxed{V_{BC} = 0}$ (at same point)

Q.NO.2

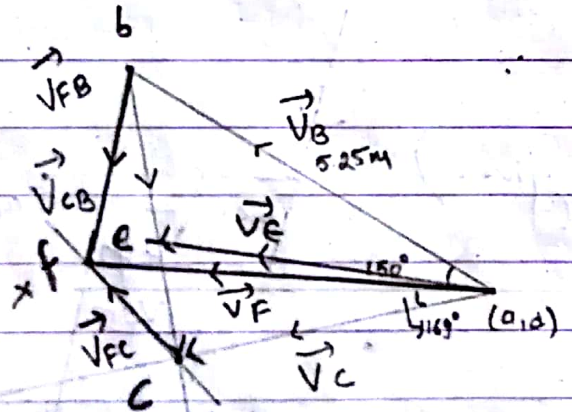
In a 4 bar mechanism AB = 50mm, BC = 66mm, CD = 56mm, AD = 100mm. At the instant when ∠DAB = 60°, the link AB has an angular velocity of 10.5 rad/s in ccw (counter clockwise). Det^n the
i) velocity of the point 'c'. ii) Vel. of point E on BC, BE = 40mm
iii) Ang. vel of BC & CD iv) Vel. of an offset point F on BC if BF = 45mm
CF = 30mm & BCF is read clockwise.

Solution:-



Configuration diagram Fig — (a)

Vector diagram:



Scale 1:100

or, $\boxed{5.25m = 5.25cm}$

Vector Diagram

fig (b)

Processes

Step: 1: Drawing the figure with given values using ruler, compass & protector.

For q. (ii) point E can be find since, BE = 40mm

q. (iv) Offset point F can be in outside of polygon or inside but in question BCF is given as clockwise so, looking to figure, for clockwise F must be inside polygon

Step 2:- For vector diagram

a) Draw refrence line & mark point (a,d). [Here, A & D both are fixed]

b) Draw $ab \perp AB$. Here direction of $\perp^{rd}$ line of B is same to the angular vel. direction. (↶, ↶)

From point (a,d) draw an line of 5.2m with an angle of $(60°+90°) = 150°$ [$\perp^{rd}$ line is drawn so add 90°]

c) Draw $\perp^{rd}$ from C, in Vec. Dia. taking origin $\vec{BC}$ (from B) (draw a line with an angle of $(50°+90° = 130°)$ (see below reading of protector)

d) Draw $\perp^{rd}$ from C w.r.t. CD line in vector diagram (similar to process b) draw a line of angle $(79°+90°)$ from (a,d) pt. taking refrence xy & intersect at c.

For calculation

(*). we have to find vel. of point C so,

$V_{CB} + V_{BA} = V_{CA}$ or,

$V_{CB} + V_{BA} = V_{CD}$. (Here, $V_{CA} = V_{CD}$)

Given, $\omega_{AB} = 10.5\ rad/s$

$AB = 50mm = 0.05m$

So, $V_{AB} = \omega_{AB} \cdot AB$

$V_{AB} = 0.05 \times 10.5 = 0.525\ m/s$

Now, we make a $\boxed{\text{Scale factor of } 1/10}$ (1/10)

i.e. $V_{AB} = 0.525 \times 10 = 5.2m$

So, $\boxed{0.525\ m/s = 5.25m}$

look to next question, here we also don't need scale factor (1m/s = 1cm)

e) Measure $\vec{V}_B$, $\vec{V}_C$

we have,

Dst. ab = 5.25 m

Dst. ac = 4.1 m

Dst. bc = 3.5 m

To convert into velocity (multiply by scale factor $= \frac{1}{10}$) or divide by 10

$\vec{V}_{AB}$ or $\vec{V}_B$	=	$5.25 \div 10$	= 0.525 m/s
$\vec{V}_{AC}$ or $\vec{V}_C$	=	$4.1 \div 10$	= 0.41 m/s
$\vec{V}_{CB}$	=	$3.5 \div 10$	= 0.35 m/s

∴ (i) Velocity of C B 0.35 m/s

for q. (ii). (Calculation)

Given Be = 40 mm

So, $\frac{Be}{Bc} = \frac{40}{66}$

We know,

$$\frac{BE}{BC} = \frac{be}{bc}$$

or, $\frac{0.040}{0.066} = \frac{be}{0.35}$

or, be = 2.1 m (Plot in vector diagram)

(iii). Velocity of BE ($\vec{V}_{BE}$) = $2.1 \div 10 = 0.21 m/s^2$

But we should find $\vec{V}_{EA} = ?$

By measure

EA (ea) = 4.2 m

∴ $\vec{V}_E = \vec{V}_{EA} = 4.2 \div 10$

$= 0.42 m/s$

for question (III)

Angular vel. of BC & CD

$\omega_{BC} = \frac{V_{BC}}{BC}$ $\qquad$ $\omega_{CD} = \frac{V_{CD}}{CD}$

$= \frac{3.5}{0.66}$ $\qquad$ $= \cancel{\frac{4.1}{0.41}} \quad \frac{4.11}{0.56}$

$= \underline{5.3 \text{ rad/s}}$ $\qquad$ $= \underline{7.3 \text{ rad/s}}$

For question (IV)

Find point F using compass (CF = 30mm, BF = 45mm)

Draw ⊥rd from F w.r.t B & w.r.t C resp.

In vector diagram :-

F point lies outside bc because BCF must be in clockwise dire. (of Given).

a) Draw parallel FM to bf. or, make an angle of 73° from pt. b taking refrence at ba [because in fig. a) ∠ABF = 73° & for parallel ∠abf = 73°]

b) Draw bc similarly, (∠DCF = 56°, so, ∠acf = 56°)

$V_{FA} = V_{FD}$

$$\boxed{V_{FB} + V_{BA} = V_{FC} + V_{CD}}$$

measuring distance af by joining them we get

Dis. af = 5.2m

$\therefore \vec{V_F} = 5.2 \div 10$

$\vec{V_F} = \underline{0.52 \text{ m/s}}$ Ans.

Q. NO. 3

An instantaneous configuration of a 4-bar mechanism whose plane is horizontal as shown in fig. At this instant the angular vel. and angular acceleration of link O_2A are $\omega = 8$ rad/s and $\alpha = 0$, resp. and the driving torque (τ) is zero. The link O_2A is balanced so that its centre of mass falls at O_2.

Find

(i). ω of link BO_4

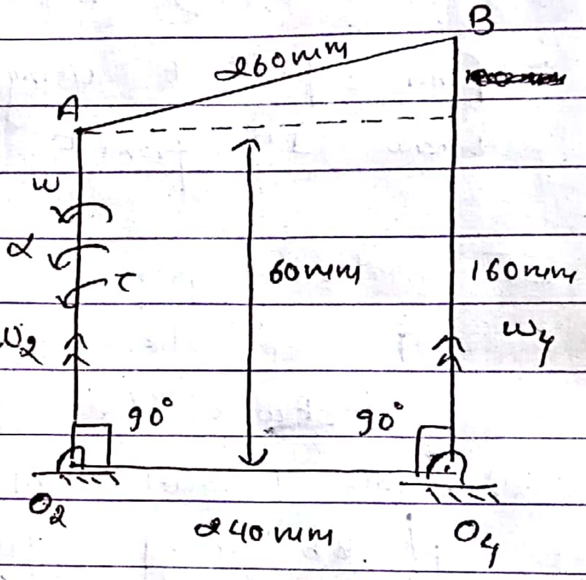
(ii). What kind of 4-bar mechanism is O_2ABO_4?

a) Double crank mechanism

b) " rocker "

c) Crank "

d) Parallelogram "



Q. (ii) Sol^n:

Using Grashof's law

$$l + s < p + q$$

$$260 + 60 < 160 + 240$$

$$320 < 400$$

Since $l + s < p + q \Rightarrow$ So, C-R mechanism (s is a crank)

Q (i) Sol^n:- (Since they are parallel)

$$\vec{V_{AO_2}} = \vec{V_{BO_4}}$$

$$\omega_2 \times AO_2 = \omega_4 \times BO_4$$

or, $$8 \times 60 = \omega_4 \times 160$$

$$\boxed{\omega_4 = 3 \text{ rad/s}}$$ angular vel. of BO_4

$V_{BA} = 0$ (as stated in Q No. 1, look a page before)

Velocity diagram of slider-crank mechanism

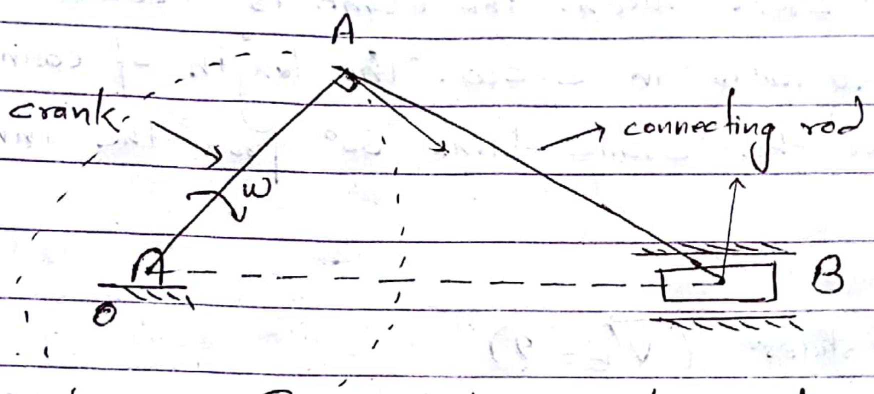


Fig: Slider crank mechanism

Vector Diagram of slider crank mechanism:

Steps:-

(i). Make a refrence point (O, B) and a line ob as both are fixed link

(ii). Draw oa ⊥ OA (since (↷) ω so, (→) ⊥rd line goes tow direc")

(iii). Draw ab ⊥ AB (Draw ⊥rd from B and project to vel. dia)

(iv). Draw ob which is horizontal line as B reciprocates in a horizontal line

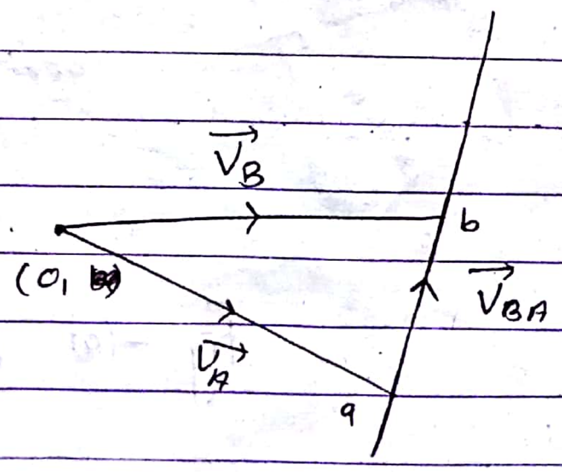
In question:

$V_A = \omega_A \cdot OA$

$\omega_{AB} = ?$

$V_{AB} = ?$

$V_B = ?$



we have,

$V_{AB} = \omega_{AB} \cdot AB$

$\omega_{AB} = \dfrac{V_{AB}}{AB}$

$= \dfrac{ab}{AB}$

= Ans. rad/sec (↺) (B बाट A मा हेर्दा anticlockwise)

- Arjun Gautam

Numerical of slider crank Mechanism:-

Q. In A slider crank mech. the crank is 480mm long and rotates at 20 rad/s in c.cw. The length of connecting rod is 1.6m. When the crank turns 60° from the inner dead centre.

Detn:

i) Velocity of slider ($\vec{V}_B = ?$)

ii) Vel. of point E located at a distance of 450mm on the connecting rod extended

iii) Position & vel. of pt. F on the connecting rod having the least absolute vel.

iv) Angular vel. of connecting rod.

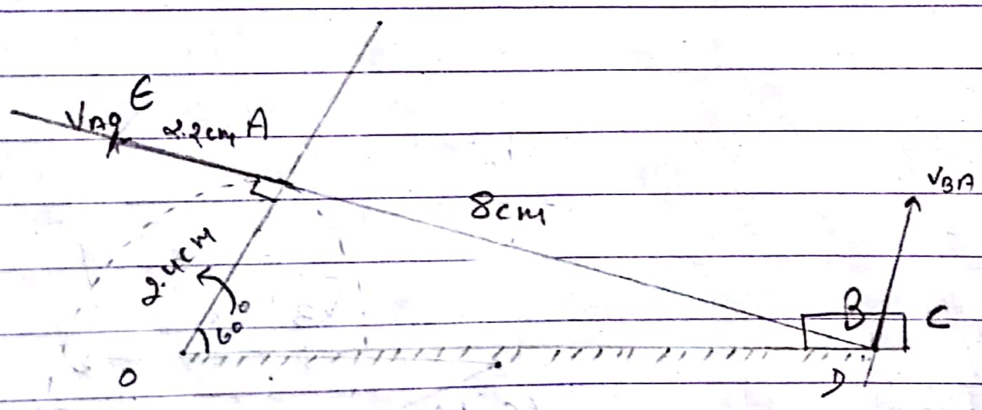


Fig -(a) Configuration diagram

Scale :- 1cm = 200mm

$480mm = \frac{480}{200} = 4.8cm$

$\therefore$ 1:200

$1600mm = \frac{1600}{200} = 8cm$

$AE = 450mm$

$= \frac{450}{200} = 2.25cm$

Velocity diagram

Steps:

a) mark a point (o,d) & from o draw a line of 9.6cm with an angle of 60°.

b) From point a draw a line of angle 77°

c). From point (a,d) draw a line parallel to OD (i.e. a horizontal line upto that point where it intersect)

Calculation

$V_{BO} = V_{BA} + V_{AO}$
$V_{BD} = V_{BA} + V_{AD}$ } (same)

$\omega_{OA} = 20 \text{ rad/s}$

$OA = 480 \text{ mm} = \cancel{480}\ 0.48 \text{ m}$

$V_{OA} = \omega_{OA} \cdot OA$
$= 20 \times 0.48$
$= 9.6 \text{ m/s}$

we have,

$9.6 \text{ m/s} = 96 \text{ mm or } 9.6 \text{ cm}$

Scale factor = $\frac{1}{10}$ if we measure in m

Since, $9.6 \text{ m/s} = 9.6 \text{ cm} = 0.96 \text{ m}$

But for $9.6 \text{ m/s} = 9.6 \text{ cm}$
or, $1 \text{ m/s} = 1 \text{ cm}$

$\boxed{\text{No need of scale factor}}$

From vel. Dia:

$oa = 9.6 \text{ cm}$
$ab = 5.1 \text{ cm}$
$ob = 9.7 \text{ cm}$

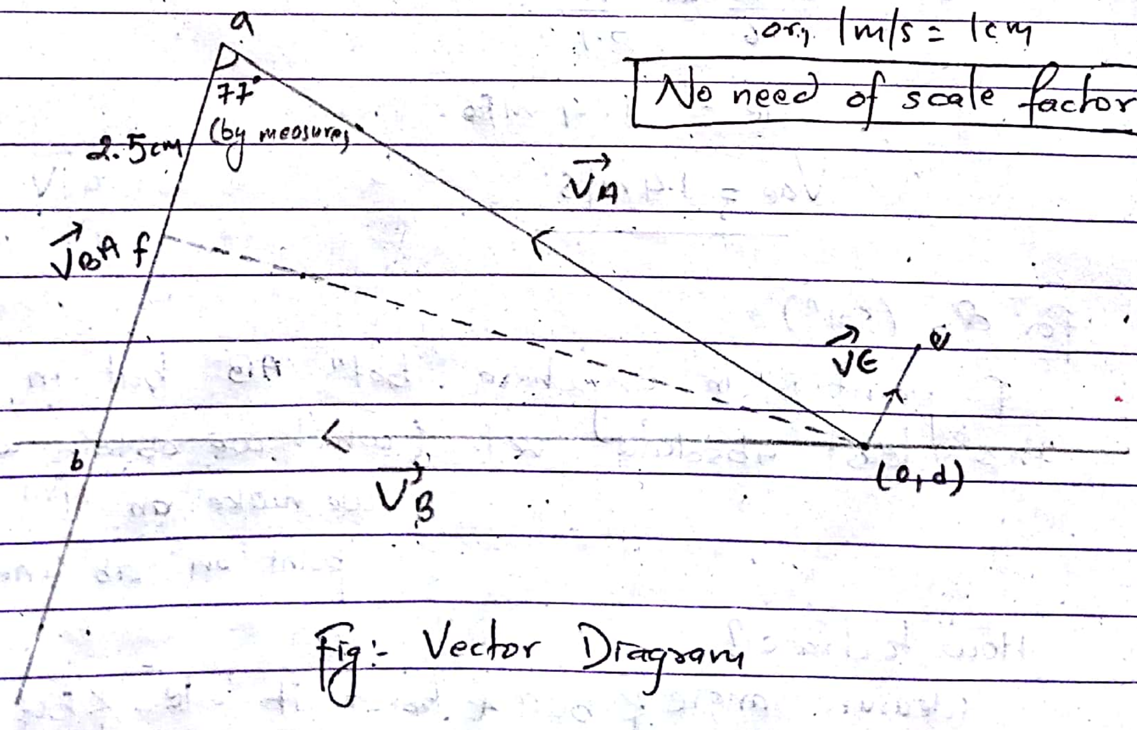


Fig:- Vector Diagram

$oa = 9.6 cm$

$ab = 5.1 cm$

$ob = 9.7 cm$

$\therefore \vec{V_A} = 9.6 m/s$

$\therefore$ Q1 soln: $\vec{V_B} = 9.7 m/s$

For Q2 sol.

extended 450mm (AE = 450mm)

$$\frac{AE}{AB} = \frac{450}{1600}$$

or, $$\frac{AE}{AB} = \frac{ae}{ab}$$

or, $$\frac{450}{1600} = \frac{ae}{5.1}$$

or, $$\frac{0.45}{1.6} = \frac{ae}{5.1}$$

$$ae = 1.4 m$$

$\therefore V_{ae} = 1.4 m/s$

for Q3 (soln)

F point B in anywhere betn AB but in question it state the least absolute vel. (which we obtain when in vel-diagram we make an $\perp^{rd}$ from point o to any point on ab line)

How to draw?

Measure angle ∠boa & bisect it ie ∠bof = 15° & draw.

Now,

$$\frac{AF}{AB} = \frac{af}{ab}$$

or, $$\frac{2.5}{5.1} = \frac{AF}{1.6}$$

$AF = 0.78 \approx 0.8 m$

Now,

measure of $of = 9.4 cm$

$\vec{V_F} = 9.4 m/s$ ($1cm = 1m/s$)

for Q.(4)

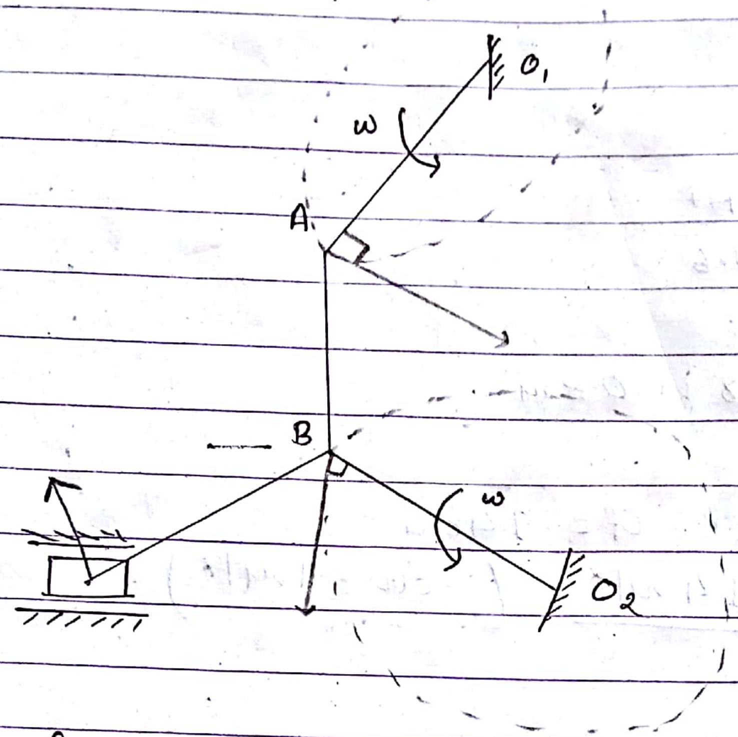
$V_{AB} = 5.1 m/s$

$$\omega_{AB} = \frac{V_{AB}}{AB}$$

$$= \frac{5.1}{1.6}$$

$\omega_{AB} = 3.18 \ rad/s$

Q. Draw a velocity diagram of following figure.



Process

(I). Mark refrence (O_1, O_2)

(II). Draw $O_1a \perp OA$

(III). Draw $O_2b \perp O_2B$

(IV). Draw $ab \perp AB$

(V). Draw $bc \perp BC$

(VI) Draw (O_1c, O_2c) horizontal

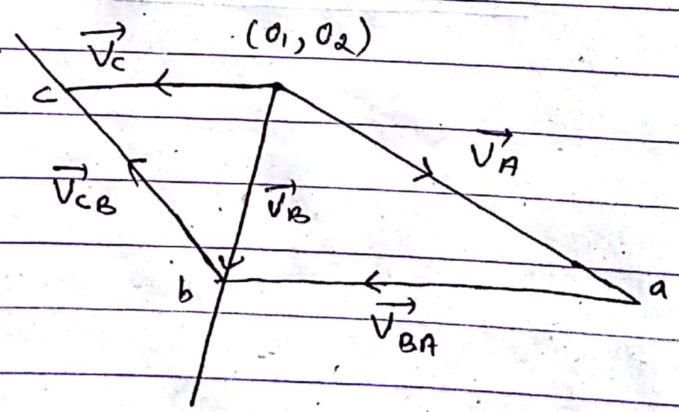
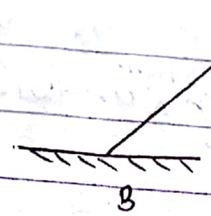


Fig: Vector Diagram

Acceleration Diagram :-



Drawing acceleration diagram, two component play an important role

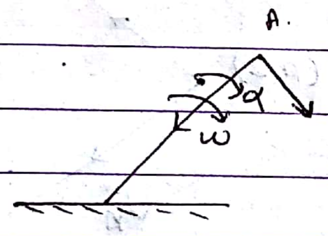
① radial component
② tangential component

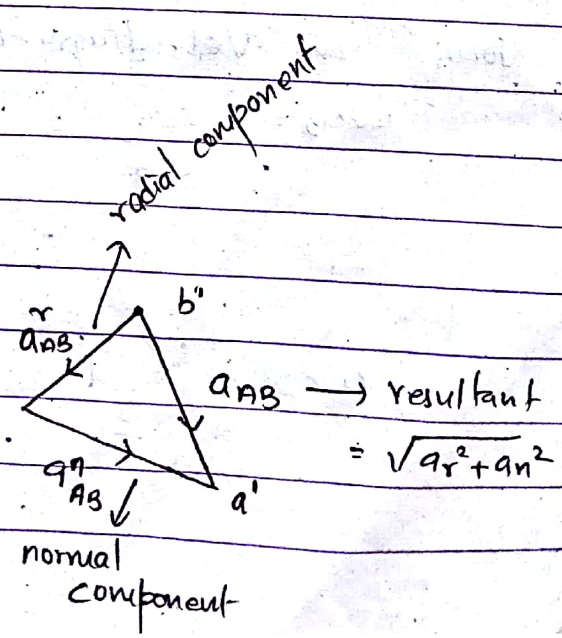
① radial component :- parallel to link or perpendicular to velocity

$$= \frac{V_{AB}^2}{AB}$$

② tangential component : perpendicular to link or parallel to velocity

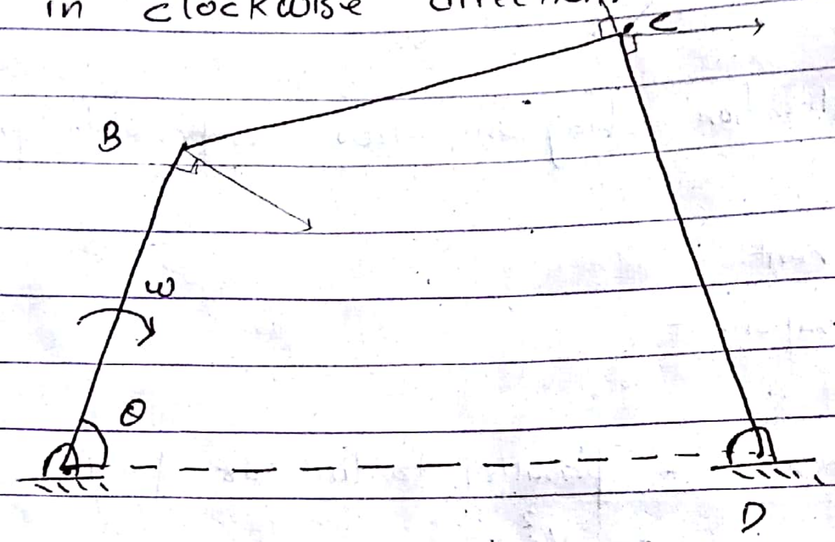
$$= \alpha \times AB$$





Acceleration diagram of four bar Mechanism

Consider a four bar mechanism as shown in figure in which crank AB rotating with non uniform angular velocity ω in clockwise direction.



Given values:-

ω = Given
θ = "
AB = "
BC = "
CD = "
AD = "

To find angular velocity ω_{BC} & ω_{CD}

Draw velocity diagram

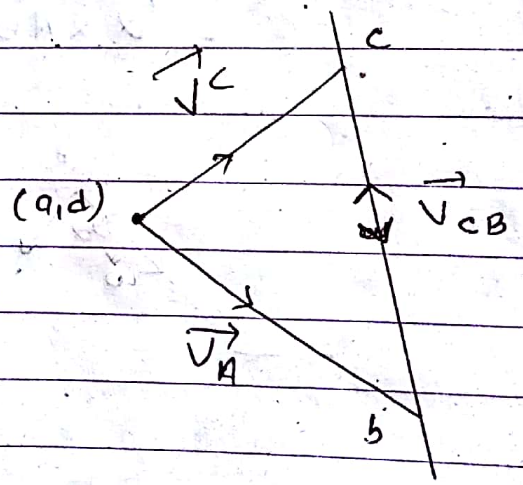
Steps

i) $ab \perp AB$
ii) $bc \perp BC$
iii) $cd \perp CD$

As, ω_{AB} & AB is given so,

$V_B = \omega_{AB} \cdot AB$ (found)

$= \text{---}$ m/s



Now, from Vel. diagram

$$\omega_{CB} = \frac{V_{CB}}{CB} = \frac{cb}{CB}$$

$= \text{....}$ rad/s (↻)

Vector diagram

&

$$\omega_{CD} = \frac{cd}{CD} = \text{....} \text{ rad/s } (\curvearrowright)$$

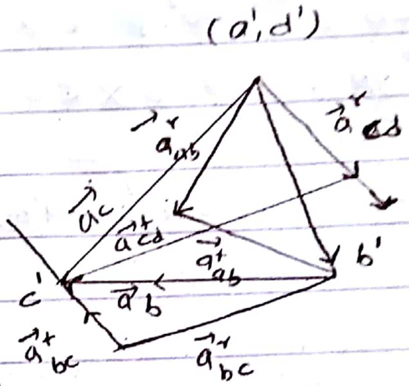
c को fixed manne
B or D भी हो

Diagram for non-uniform angular velocity

For acceleration:-

Steps

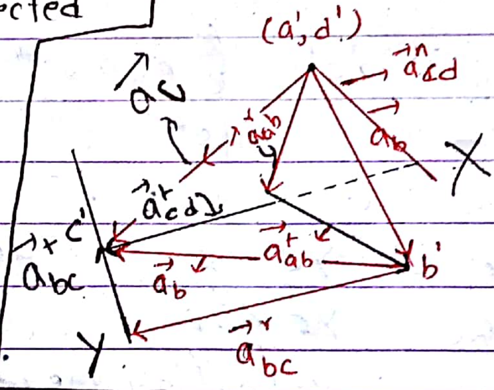
1. Mark refrence point (a', d')
2. Draw $AB \parallel \vec{a}^{r}_{ab}$
3. Draw $ab \parallel \vec{a}^{t}_{ab}$
4. Joint $\vec{a}^{r}_{ab}$ & $\vec{a}^{t}_{ab}$
5. Again Draw $BC \parallel \vec{a}^{r}_{bc}$ from b'
6. Draw $bc \parallel \vec{a}^{t}_{bc}$ (unknown) so,
7. Draw $CD \parallel \vec{a}^{r}_{cd}$
8. Draw $cd \parallel \vec{a}^{t}_{cd}$ (which intersect at a line of $bc \parallel \vec{a}^{t}_{bc}$ & joint it)
9. Draw resultant line betn $\vec{a}^{r}_{bc}$ & $\vec{a}^{t}_{bc}$
10. Draw a line from (VIII) intersected point to (a', d') which is $\vec{a}_c$
11. Completed //



For understanding

red pen = radial component + resultant

black pen: tangential



Acceleration Diagram

Note:-

If uniform velocity is given then tangential component will be zero but not radial (normal) component.

Calculation:-

For link AB

$a_B = a^{r}_{AB}$ $\quad$ ($a_t = 0$ for uniform case)

$= \omega^2 . AB$ or $\frac{V_{AB}^2}{AB}$ if given)

$\therefore \ldots\ m/s^2$

for link CD

$a^r_{CD} = \omega^2_{CD} \cdot CD$ (mg. dir^n known)

$a^t_{CD} = \alpha_{CD} \times CD$ (unknown)

$a_c = \sqrt{{a^r_{CD}}^2 + {a^t_{CD}}^2}$

for link BC

$a^r_{BC} = \omega^2_{BC} \cdot BC$

$\ldots$ (known)

$a^t_{BC} = \alpha_{BC} \times BC$ (α_{BC} = unknown) —— (I)

$\vec{a}_{BC} = b'c'$ (in accl^n diagram)

$= \ldots$ m/s²

$a^t_{BC} = YC' \times SF$ (in accl^n dia.) SF = scale factor

$\alpha_{BC} \times BC = YC' \times S.F$

$\alpha_{BC} = \dfrac{YC' \times S.F \text{ (by measure)}}{BC \text{ (given)}}$ —— (II)

$= \ldots$ m/s² (↺)

Put value of eq^n (II) in eq^n (I)

$a^t_{BC} = \alpha_{BC} \times BC$ (known)

& $a_b = \sqrt{{a^r_{BC}}^2 + {a^t_{BC}}^2}$ (known)

$= \ldots$ m/s²

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Or, using acceleration diagram

Absolute accln of point c

$a_c = a'c' \times$ scale factor

$= \ldots\ldots$ m/s²

$a^t_c = xc' \times S.F$

$\alpha_{CD} \times CD = xc'$

$$\alpha_{CD} = \frac{xc' \times SF}{CD}$$

$= \ldots\ldots$ rad/sec² (↻)

$a_b = a'b' \times SF$

$= \ldots\ldots$ m/s²

How to find angular acceleration direction:-

(*). See tangential comp. direction

(*). look to the acceleration diagram, for direc of α_{BC} watch ~~Bx~~ yc' and move (place) to c' (means ↻) of configuration diagram. Place x at c & move the line towards A so, direc is anticlockwise

(*). for direction of α_{CD}, place c'x in point c of configuration diagram. Here, the direction is inward (←) so, the line c'x moves inward of configu. diag. from c & look or rotate the line to D it is anticlockwise again.

Numerical for acceleration:-

Q. PQRS is a four bar chain with link PS fixed. The lengths of the link are PQ = 62.5mm ; QR = 175mm ; RS = 112.5mm ; and PS = 200mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagram when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of link QR and RS.

Soln: Given,

PQ = 62.5mm , QR = 175mm , RS = 112.5mm , PS = 200mm

PQ (ω_{PQ}) = 10 rad/s (↻) , ∠QPS = 60°

Scale : 1cm = 20mm

1:20

(200mm = 10cm)

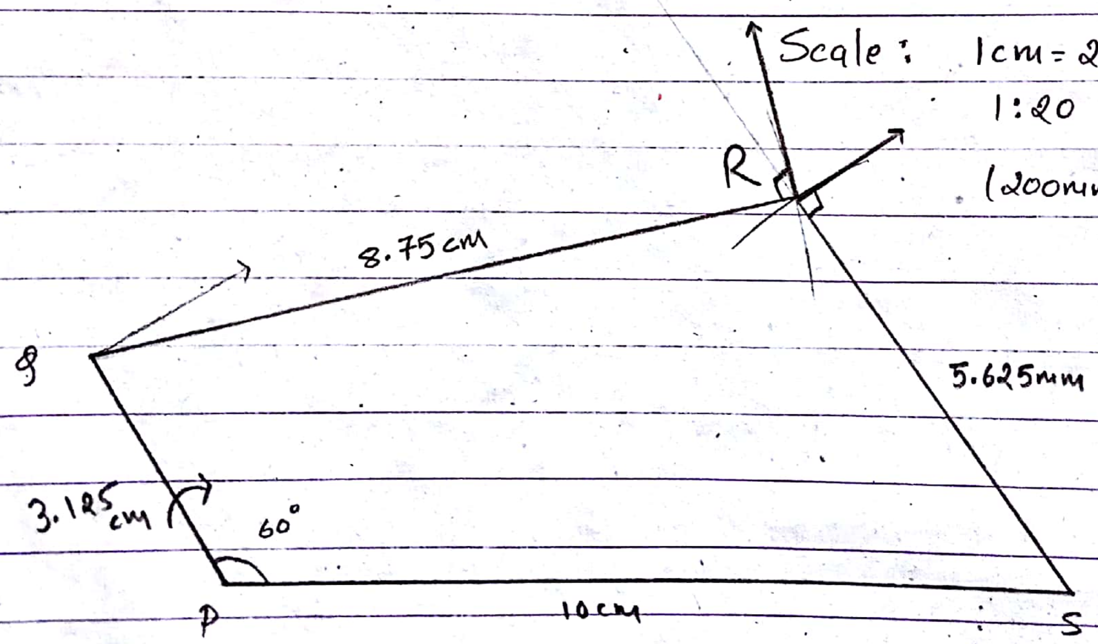


Fig:- Configuration diagram

$V_{PQ} = \omega_{PQ} \times PQ$

$= 10 \times \frac{62.5}{1000}$

$= 0.625 m/s$

Scale: 1 m/s = 0.1m

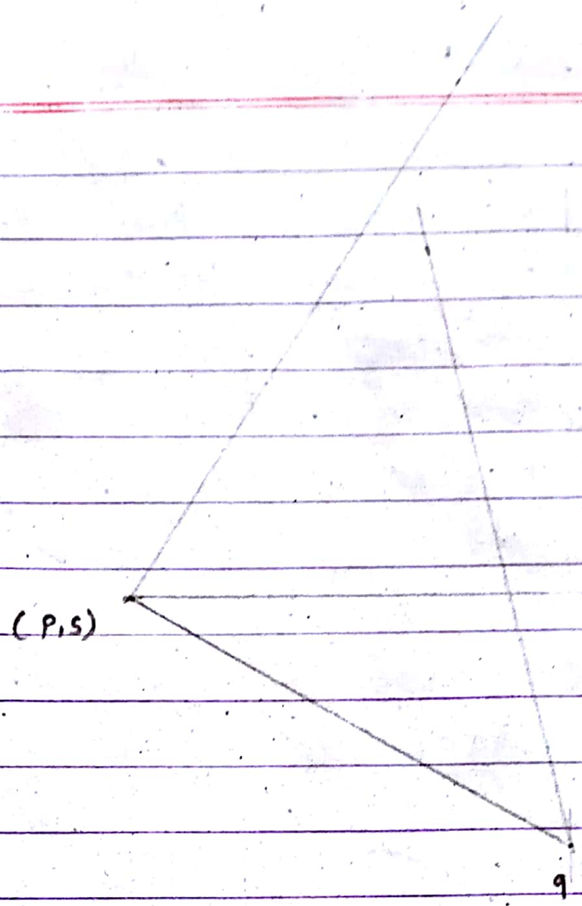
0.625 m/s = 0.625 m × 0.1

= 0.0625m

= 6.25cm

$\boxed{\text{Scale factor : 0.1}}$

Vector diagram



Q. Numerical for acceleration: (Theory of machine Page No. 184)

PQRS is a four bar chain with link PS fixed. The lengths of the links are PQ = 62.5mm; QR = 175mm; RS = 112.5mm, and PS = 200mm. The crank PQ rotates at 10 rad/s clockwise. Draw the velocity and acceleration diagrams when angle QPS = 60° and Q and R lie on the same side of PS. Find the angular velocity and angular acceleration of links QR and RS.

Solution:-

PQ = 62.5mm, QR = 175mm, RS = 112.5mm, PS = 200mm

ω_{PQ} = 10 rad/s $\angle QPS = 60°$

Now,

$$V_{PQ} = \omega_{PQ} \cdot PQ$$

$$= 10 \times \frac{62.5\,mm}{1000\,mm} \times 1m$$

$$= 0.625\ m/s$$

Now,

$\boxed{\text{Scale factor} = 0.1}$

0.625 m/s = 0.625 × 0.1 (m)

= 0.0625 m

= 6.25 cm

Fig:- Configuration diagram

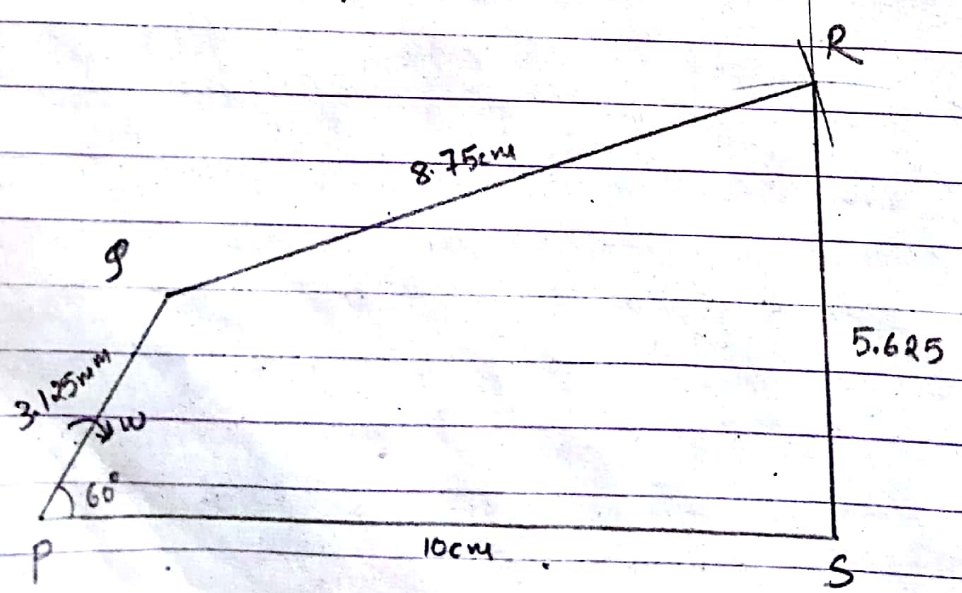


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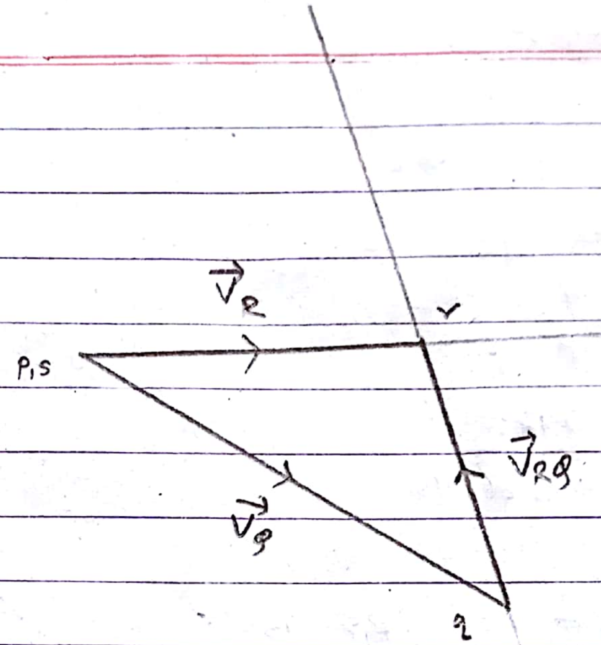
Fig: Vector Diagram



Now,

from vel. Dig. so,

pr = 4.3 cm $\quad V_R = 4.3 \times 0.1 = 0.43$ m/s

rq = 3.4 cm $\quad V_{RQ} = 3.4 \times 0.1 = 0.34$ m/s

pq = 6.25 cm $\quad V_Q = 6.25 \times 0.1 = 0.625$ m/s

Also,

$$\omega_{SR} = \frac{V_R}{SR} = \frac{0.43}{0.1125} = 3.8 \text{ rad/s (clockwise)}$$

$$\omega_{QR} = \frac{V_{QR}}{QR} = \frac{0.34}{0.175} = 1.9 \text{ rad/s (Anticlockwise)}$$

Calculation of angular acceleration.

In Question the angular accel^n (α) of link PQ is not given and α is proportional to tangential component. Hence, there will be no tangential component. Only radial (normal) component exists.

$$\vec{a}_Q = a^r_{QP} = \frac{V^2_{QP}}{QP} = \frac{(0.625)^2}{0.0625} = 6.25 \text{ m/s}^2$$

Acceleration diagram:-

Also,

Radial component of QR is

$$a^r_{RQ} = \frac{V^2_{RQ}}{RQ}$$

$$= \frac{(0.34)^2}{0.175}$$

$$= 0.66 m/s^2$$

Scale factor

$\boxed{1 m/s^2 = 1 cm}$

Also,

Radial component of RS is

$$a^r_{RS} = \frac{V^2_{RS}}{RS}$$

$$= \frac{(0.43)^2}{0.1125}$$

$$= 1.64 m/s^2$$

$\boxed{\text{No need of scale factor.}}$

Now,

the acceleration diagram on next page is drawn as follows:-

Process:- Process listed below were the process of making diagram using angle method due to insufficient paper size.

Steps:-

1) Mark p's' at any point / line.

radial 2) Draw p'q' || PQ (an angle of 60° & a^r_{PQ} = 6.25 cm)

radial 3) Draw q'x || QR (an angle of 41° from q' & q'x = 0.66 cm)

tangential 4) Draw xr' || qr (vel. dia. ⊥) [make 90° ∡ from x & xr' = unknown]

radial 5) Draw s'y || RS (making an angle of 88° from p' & dist. ys' = 1.64 cm)

tangential 6) Draw a ⊥rd line from y towards xr' line (similar to process 4)

finding r' 7) Intersect the line which will be r'

finding $\vec{a}_{rq}$ 8) Draw a dark line from r' joining q' (r'q')

finding $\vec{a}_r$ 9) Similarly, draw a dark line from r' to (p's') point

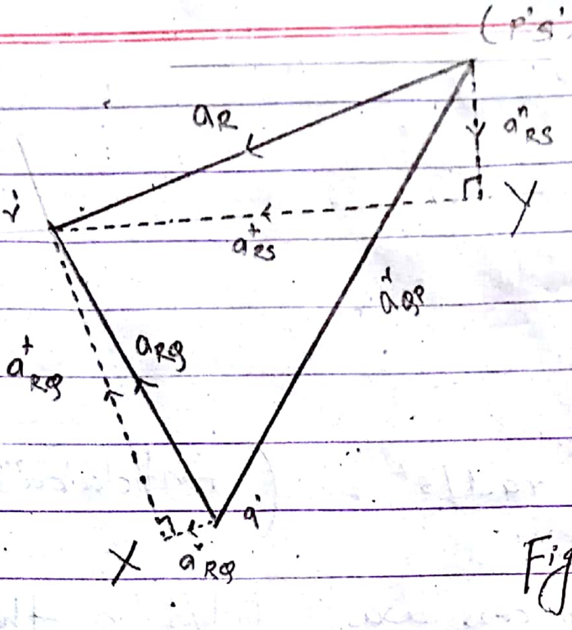


Fig:- Acceleration Diagram

From acceleration diagram :-

$p'y =$	1.64 cm	$a^n_{RS} =$	1.64 m/s²
$yr' =$	5.2 cm	$a^t_{RS} =$	5.2 m/s²
$r'p' =$	5.4 cm	$a_R =$	5.4 m/s²
$p'q' =$	6.25 cm	$a^r_{QP} =$	6.25 m/s²
$q'r' =$	4.1 cm	$a_{RQ} =$	4.1 m/s²
$q'x =$	0.66 cm	$a^r_{RQ} =$	0.66 m/s²
$xr' =$	4.0 cm	$a^t_{RQ} =$	4 m/s²

Calculation:

Angular accⁿ of QR

$$a^t_{RQ} = \alpha_{QR} \times RQ$$

$$\therefore \alpha_{QR} = \frac{a^t_{RQ}}{RQ}$$

$$= \frac{4.1}{0.175}$$

$$\therefore \alpha\, 23.43 \text{ rad/s}^2 \quad \text{(Anticlockwise)}$$

Angular acceln of RS

$$a^t_{RS} = \alpha_{RS} \times RS$$

or, $$\alpha_{RS} = \frac{a^t_{RS}}{RS}$$

$$= \frac{5.2}{0.1125}$$

$$= 46.22 \text{ rad/s}^2 \quad \text{(Anticlockwise)}$$

Note:-

Direction finding process was listed in theory part pls turn previous pages

Acceleration diagram for Slider Crank Mechanism :-

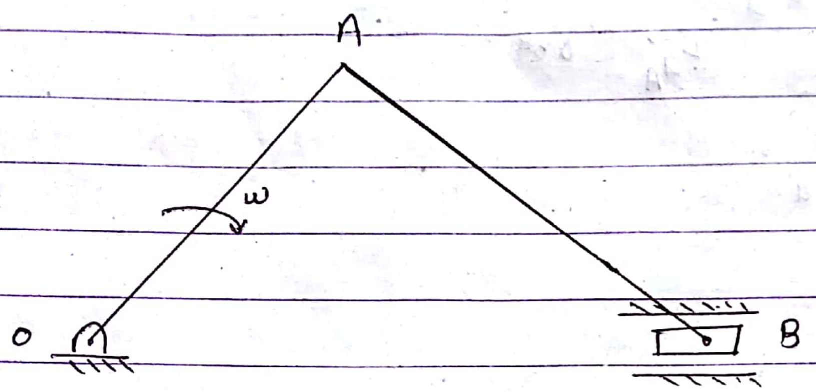


Fig :- Slider crank mechanism

Crank OA rotates with uniform angular velocity ω in clockwise direction.

Velocity Diagram :- (As before)

Steps

Draw.

i) oa ⊥ OA

ii) ob, horizontal line through pt. o

iii) ab ⊥ AB

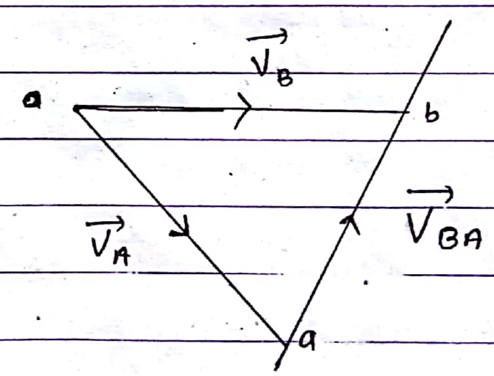


Fig :- Vel. Dig.

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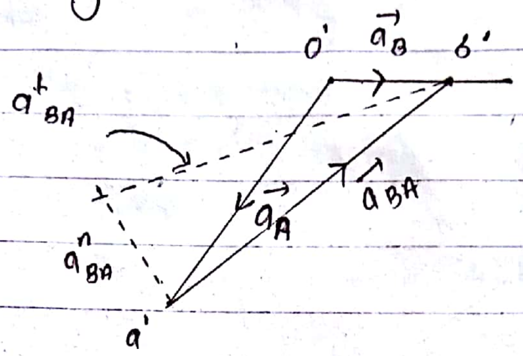
Acceleration Diagram:-



Processes:-

Steps:-

(I) $o'a' \parallel OA$ (radial component)

(II) $a'x \parallel AB$ (radial component)

(III) $xb' \perp AB$ or $xb' \parallel ab$ (tangential component)

(IV) $ob' \parallel OB$ (radial + tangential component)

Numerical: (Theory of machine Page No. (77))

The crank of a slider crank mechanism rotates clockwise at a constant speed of 300 rpm. The crank is 150 mm and connecting rod is 600 mm long. Determine Linear vel. & angular vel. also

1. Acceleration of the mid-point of the connecting rod
2. Angular acceleration of the connecting rod, at the crank angle of 45° from inner dead centre position.

Solution:-

$OB = 150mm = 0.15m$

$BA = 600mm = 0.6m$

$N_{BO} = 300rpm$

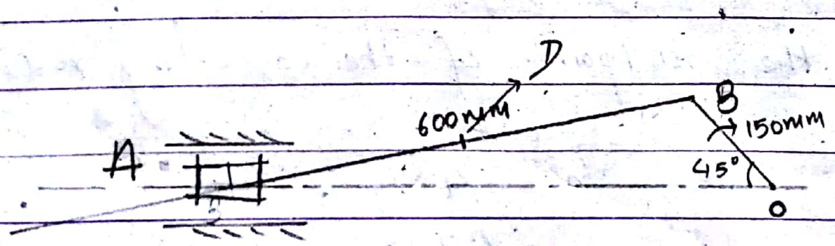
$\omega_{BO} = \frac{2\pi N}{60} = \frac{2 \times \pi \times 300}{60} = 31.42 \text{ rad/sec.}$

$\angle AOB = 45°$

Fig:- Configuration or Space Diagram:

Scale:	
600mm	= 6cm
150mm	= 1.5cm

Steps:- Use compass & ruler for drawing



We know, linear velocity of OB

$$V_{OB} = \omega_{OB} \times OB$$

$$= 31.42 \times 0.15$$

$$= 4.713\ m/s$$

Scale : $4.713\ m/s = 4.713\ cm$

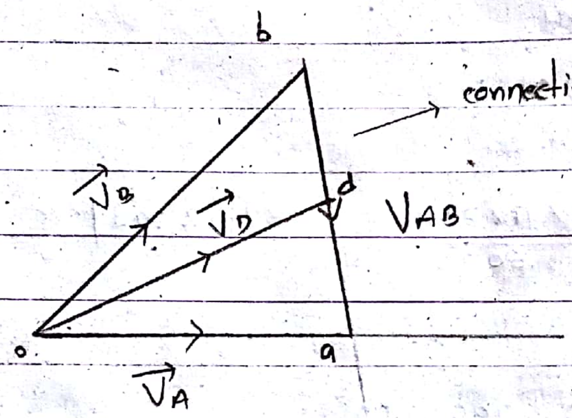


Fig :- Velocity Diagram

Steps:-

(I) marks point o

(II) Draw ob ⊥ OB (an angle of 135° (90+45) from o)

(III) ba ⊥ BA (an angle of 125° with unknown dst)

(IV) oa horizontal line intersecting a line from b.

(1) Linear velocity of the midpoint of the connecting rod:-

By measurement:-

oa = 4 cm ie. $V_A = 4\ m/s$

ob = 4.713 cm $V_B = 4.713\ m/s$

ab = 3.4 cm $V_{AB} = 3.4\ m/s$.

In order to find the velocity of mid-point D of connecting rod AB divide ba into half & mark point d. Then measure distance ad.

$$ad = \frac{3.4}{2} = 1.7 cm$$

$\therefore$ od = 4.1 cm i.e. $\vec{V_D} = 4.1 m/s$ //

Acceleration Diagram:-

for link OB

(it is moving with constant speed so there is no tangential component)

$$a^r_{OB} = a^r_{BO} = \frac{V^2_{BO}}{BO}$$

$$= \frac{(4.713)^2}{0.15} = 148.1 m/s^2$$

Also, $a^r_{AB} = \frac{V^2_{AB}}{AB}$

$$= \frac{(3.4)^2}{0.6} = 19.3 m/s^2$$

$a^r_{AO} = \frac{V^2_{AO}}{AO}$ (No need so no compute)

$$= \frac{(4)^2}{0.69} = 23.18 m/s^2$$

from above value, we can construct acceleration diagram

Scale = $1 m/s^2 = \frac{1}{20} cm$

$148.1 m/s^2 = 7.405 cm$

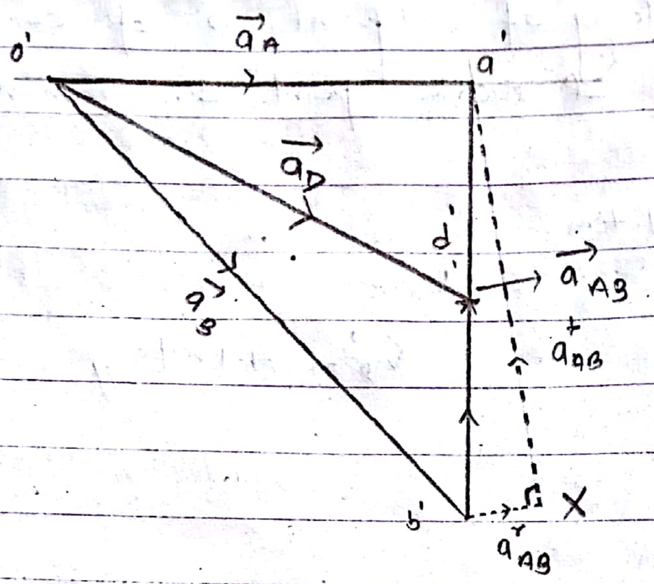


Fig: Acceleration Diagram

Process:

Steps ① Mark o'

radial (II) o'b' // OB (an angle of ($90° + 45°$) and dist. 7.405cm)

radial (III) b'x // BA (an angle of 125° and dist. 0.965cm)

tangential (IV) xa' // ab (or $\perp^{rd}$ from b'x and extend)

resultant (V) oa' is horizontal line upto intersection point

From acceleration diagram

a'x = 5.15cm	i.e. a^t_{AB} = a'x =	5.15×20 = 103 m/s^2
o'b' = 7.405cm	$\vec{a}_B$ = o'b' =	148.1 m/s^2
o'a' = 5.15 cm	$\vec{a}_A$ = o'a' =	103 m/s^2
b'x = 0.965cm	a^r_{AB} = b'x =	19.3 m/s^2
a'b' = 5.2cm	$\vec{a}_{AB}$ = a'b' =	104 m/s^2

Acceleration of midpoint of connecting rod.

i.e. a'b' = 5.2cm

$\therefore a'd' = \frac{5.2}{2} = 2.6cm$

Dist. $o'd' = 5.85$ cm i.e. $\overline{a_D} = 5.85 \times 20$

$= 117 \; m/s^2$

Q.2 Soln:

Angular vel. of connecting rod i.e. $\omega_{AB} = ?$

We have,

$$\omega_{AB} = \frac{V_{AB}}{AB}$$

$$= \frac{3.4}{0.6}$$

$$= 5.67 \; rad/s$$ (Anticlockwise)

(because put ab of vel. dia to link AB of A i.e. it is downwa[rd] & look to B, direction is anticlockw[ise])

Angular accel. of connecting rod

$\alpha_{AB} = ?$

We have,

$$\alpha_{AB} = \frac{a^t_{AB}}{BA}$$

$$= \frac{103}{0.6}$$

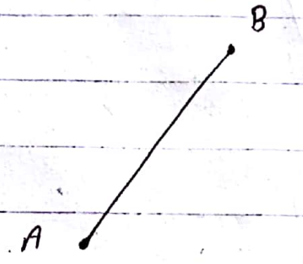
$$= 171.67 \; rad/s^2$$ (clockwise)

because take a'x & put to link AB of pt. A (as it is upward then look B) direct^n will be clockwise.

Coriolis Component of Acceleration :-

For understanding purpose only not for exam :-

Since, acceleration of point with the other point of same line is the sum of vector of tangential and radial component.



It is only possible when both point A & B are fixed i.e. (their distance is fixed).

$$a_{ba} = a^{r}_{ba} + a^{t}_{ba}$$

But if (A) point moves upward or slide upward then one more component is added i.e. (A & B dist. are not fixed)

$$a_{ba} = a^{r}_{ba} + a^{t}_{ba} + a_{cr}$$

where, a_{cr} = coriolis component of acceleration.

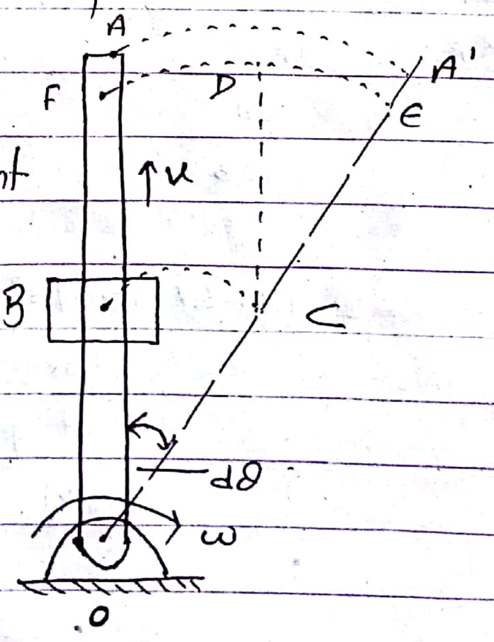


Fig:- for coriolis component

Explanation of figure:

- Link OA is rotating about point O with an angular vel. ω.
- Slider at point B, link OA rotates at angle dθ in short time dt.
- In mean time the point B slides upward (↑) with linear vel. (v).

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- In fig, the initial vel. of B is zero & final vel. is at E. We can assume that the motion of B from E is only possible when B moves to C, C to D & then D to E.

- In small time dt, the angle turn is $d\theta$ i.e. point A moves to A' & F is now taken by E.

Length of ar DE = length of arc FE − length of arc FD

$= OF \times d\theta - \text{arc } B'C$ $\left[\theta = \frac{l}{r} \text{ or, } l = r\theta\right]$

$= OF \times d\theta - OB \times d\theta$

$= (OF - OB) \times d\theta$

$= FB \times d\theta$

$= DC\, d\theta$ $[FB = DC]$

Angle is so small that we can assume it is not an arc, it is chord (i.e. linear distance).

Hence,

arc DE = chord DE (DE)

$\text{Chord } DE = DE\, d\theta$

$= v\,dt \cdot d\theta$ $\left[DC = v\,dt \text{ i.e. } v = \frac{dt}{de}\right)$

$= v\,dt \cdot \omega\,dt$ $\left[\omega = \frac{d\theta}{dt}\right]$

$\text{chord } DE = v\omega\, dt^2$ —— (I)

Slider is in initial position B i.e. its velocity $(v_B) = 0$

Distance moving with coriolis accl^n to reach B from point E is given by

$DE = u_B + \frac{1}{2} a_B t^2$

$= 0 + \frac{1}{2} a_c \times dt^2$ —— (II)

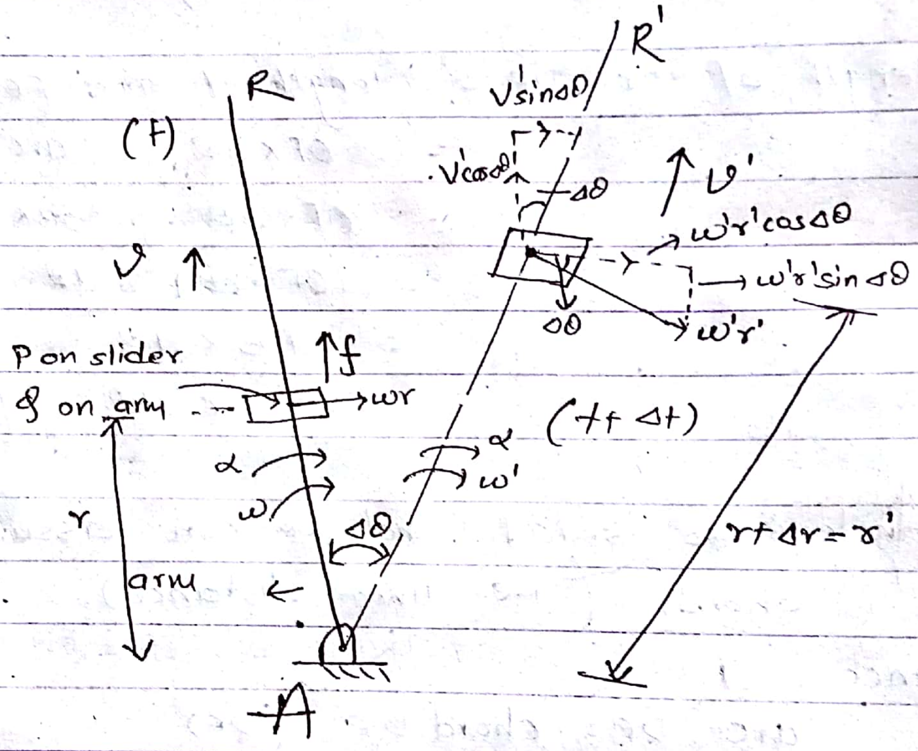
Computing eqⁿ (I) & (II), we get

$$\vartheta \omega \cancel{dt^2} = \frac{1}{2} a_c \cancel{dt^2}$$

$\boxed{a_c = 2\omega v}$ which is coriolis component of acceleration.

For Exam:-

Consider a link AR rotates about a point A link with an angular velocity ω & angular accelⁿ α & the slider slides along the link with linear vel. (v) & linear accⁿ (f) at a time (t).



Then,

$v' = v + f \cdot \Delta t \qquad \left(f = \frac{\Delta v}{\Delta t} \right)$

$\omega' = \omega + \alpha \Delta t \qquad \left(\alpha = \frac{\omega}{\Delta t} \right)$

$r' = r + \Delta r$

So, acceleration parallel to AR

change in velocity $(\Delta v) = (v'\cos\Delta\theta - \omega' r' \sin\Delta\theta) - v$

$= [(v + f\Delta t)\cos\Delta\theta - (\omega + \alpha\Delta t)(r + \Delta r)\sin\Delta\theta] - v$

Here,

$\cos\Delta\theta = 1$ (since, $\Delta\theta$ is so small $\Delta\theta \to 0$)

$\therefore \cos\Delta\theta = 1$ & $\sin\Delta\theta = \Delta\theta$

Also, neglect $\Delta\theta$.(multiple) because $\Delta\theta$ is small & if we multiply any no. we obtain more smaller value so we neglect it.

So, the eqns become

$\Delta v = v + f\Delta t - (\omega r + \omega\Delta r + \alpha r\Delta t + \alpha\Delta t\Delta r)\Delta\theta - v$

$= v + f\Delta t - \omega r\Delta\theta - \omega\Delta r\Delta\theta - \alpha r\Delta t\Delta\theta - \alpha\Delta t\Delta r\Delta\theta - v$

$= \cancel{v} + f\Delta t - \omega r\Delta\theta - 0 - 0 - 0 - \cancel{v}$

$\Delta v = f\Delta t - \omega r\Delta\theta$ ——— (I)

Now,

acceleration parallel to AR

$= \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t}$

$= \lim_{\Delta t \to 0} \left(\frac{f\Delta t - \omega r\Delta\theta}{\Delta t}\right)$

$= \lim_{\Delta t \to 0} \left(f\frac{\Delta t}{\Delta t} - \omega r\frac{\Delta\theta}{\Delta t}\right)$

$= \lim_{\Delta t \to 0} (f - \omega r.\omega)$ $\left[\omega = \frac{d\theta}{dt}\right]$

$= f - \omega^2 r$ ——— (II)

Acceleration $\perp^{rd}$ to AR

$(\xrightarrow{+})$ Change in vel. $(\Delta v) = (v'\sin\Delta\theta + \omega' r' \cos\Delta\theta) - \omega r$

or, $\Delta v = \left[(v + f\Delta t)\sin\Delta\theta + (\omega + \alpha\Delta t)(r + \Delta r)\overset{\nearrow 1}{\cos\Delta\theta}\right] - \omega r$

or, $\Delta v = (v + f\Delta t)\Delta\theta + (\omega r + \omega\Delta r + \alpha\Delta t r + \alpha\Delta t\Delta r)\overset{1}{\cancel{\cos\Delta\theta}} - \omega r$

$= v\Delta\theta + f\Delta t\cdot\Delta\theta + (\omega r + \omega\Delta r + \alpha r\Delta t + 0) - \omega r$

$= v\Delta\theta + \cancel{\omega r} + \omega\Delta r + \alpha r\Delta t - \cancel{\omega r}$

$= v\Delta\theta + \omega\Delta r + \alpha r\Delta t$

Now,

Acceleration perpendicular to AR

$$= \lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}$$

$$= \lim_{\Delta t\to 0}\left(\frac{v\Delta\theta + \omega\Delta r + \alpha r\Delta t}{\Delta t}\right)$$

$$= \lim_{\Delta t\to 0}\left(v\frac{\Delta\theta}{\Delta t} + \omega\frac{\Delta r}{\Delta t} + \alpha r\frac{\Delta t}{\Delta t}\right)$$

$$= \lim_{\Delta t\to 0}(\omega v + \omega v + \alpha r) \quad \left[\omega = \frac{d\theta}{dr},\ v = \frac{dr}{dt}\right]$$

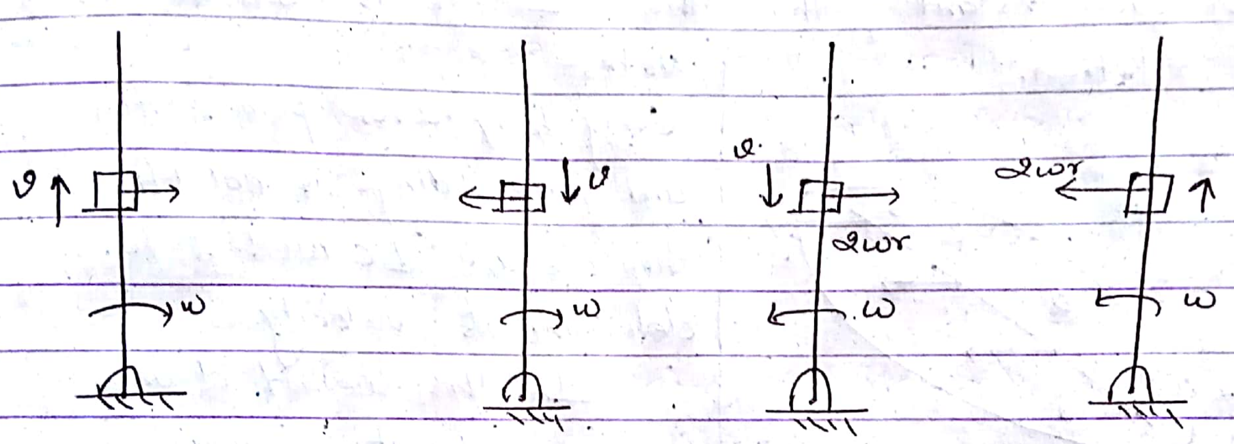
$$= \alpha r + 2\omega v$$

$2\omega v$ → coriolis component of acc^n

The direction of coriolis component $(2\omega v)$ depends on the direction of angular velocity (ω) & linear vel. (v). If ?

obtained by rotating linear velocity by 90° in the direction of angular velocity.

examples:-



How ?

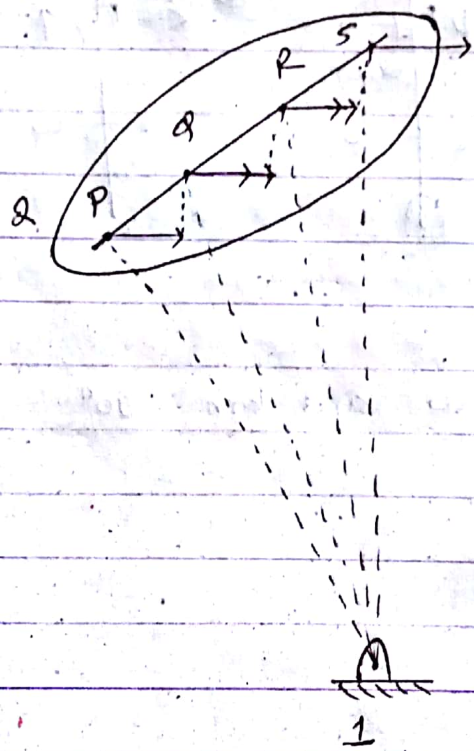
watch a v with respect to ω and turn 90° $\uparrow^{v}_{\omega}$ it turn on clockwise 90°.

Instantaneous centre method (IC) method :-

<u>Instantaneous Centre (IC) :-</u>

In this method, the motion of the kinematic link is assumed as pure rotation about a centre for given instant of time. Since, the centre of rotation of the rigid body changes for each instant, thus this centre is called instantaneous centre of rotation.

Note:-
Every body whose pure rotation and pure sliding is not defined then we use IC method to determine its velocity.
In this method at any imaginary point we assume a pure rotation & that imaginary point is called Instantaneous Centre point & process of finding a point is IC method.

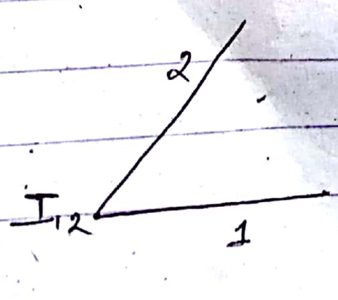


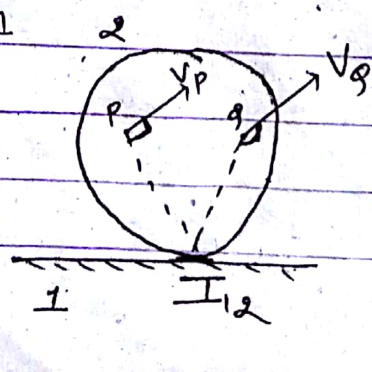
I_{21} or I_{12}

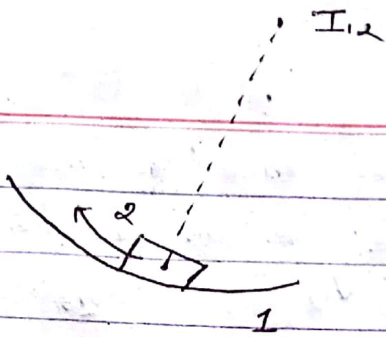
From Diagram :-

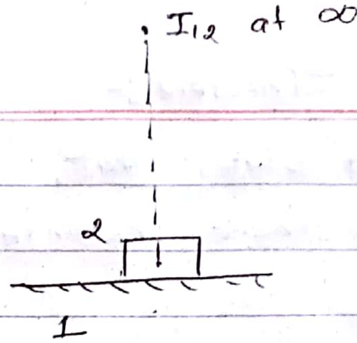
$$W = \frac{V_P}{PI_{12}} = \frac{V_Q}{QI_{12}} = \frac{V_R}{RI_{12}} = \frac{V_S}{SI_{12}}$$

<u>Instantaneous Centre Determination</u>









Number of Instantaneous centre in mechanism :-

$$\text{No. of IC} = \frac{n(n-1)}{2}$$

where, n = No. of links / elements in a mechanism

For 4 bar mechanism

$n = 4$

so,

$$\text{No. of IC} = \frac{n(n-1)}{2}$$

$$= \frac{4(4-1)}{2}$$

$$= 6$$

Types of IC

(i) Temporary IC
- material to material connection
- has different position as per position of line

(ii) Permanent IC → material to material connection and also has fixed position for any configuration of mechanism

(iii) Neither temp. nor perm. IC → No material to material connection & also has diff. location as per the configuration of mechanism.

Kennedy Theorem :-

It states that, "If three links have relative motion to each other, then their instantaneous centre lie on a straight line".

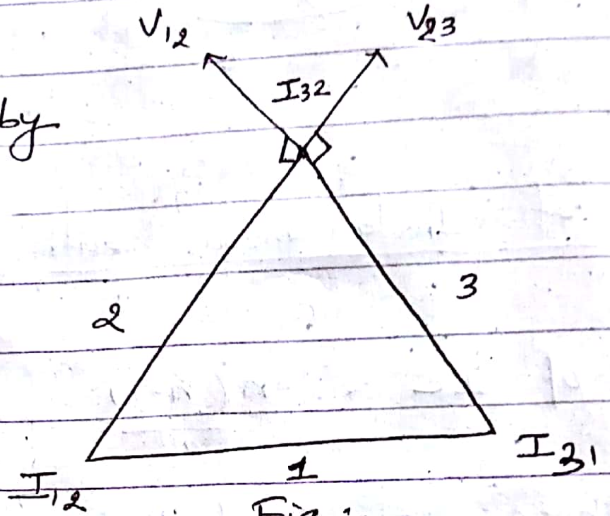


Fig :-

The no. of IC (N) is given by

$$N = \frac{n(n-1)}{2}$$

$$= \frac{3(3-1)}{2}$$

$$= 3$$

where,

The two instantaneous centre at the pin joint 2 with 1, & 3 with 1 (i.e. I_{12} and I_{13}) are the permanent instantaneous centres. According to kennedy's theorem, the third instantaneous centre I_{23} must lie on the line joining I_{12} and I_{13}.

Description of above fig:-

- Since there are 3 links & we already describe them that it is a locked mechanism.
- The direction of vel. will co-incide when angle betn. two links gradually decreased.
- When I_{32} comes in a line of I_{12} & I_{31} then their relative vel. will be zero because direction will be same.

Location of Instantaneous Centre

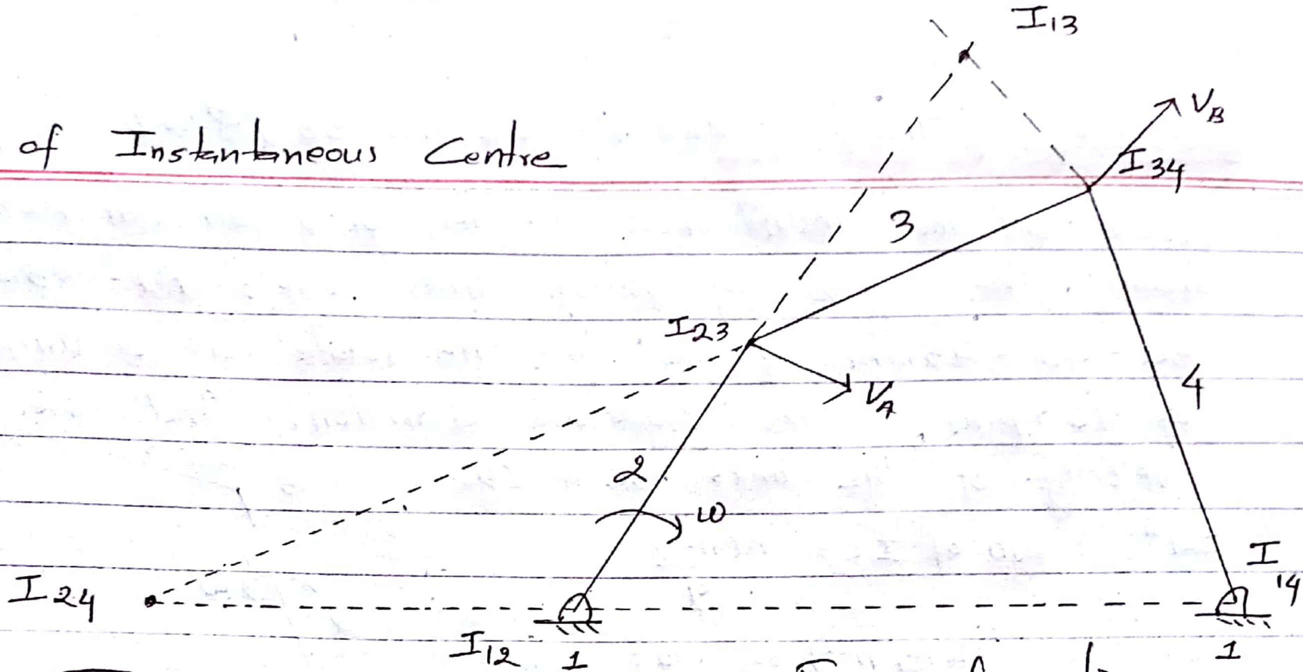
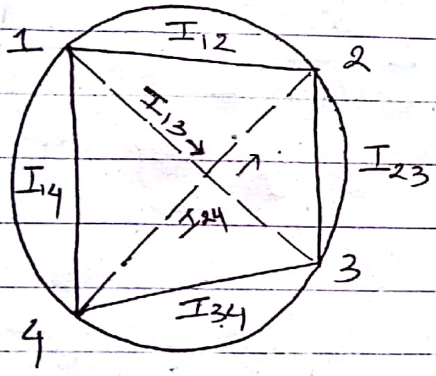


Fig:- configuration diagram



From figure:-

$$V_A = \omega \, I_{12} I_{23}$$

No. of IC $= \dfrac{n(n-1)}{2}$

$= \dfrac{4(4-1)}{2}$

$= 6$

$\therefore$ I_{12}, I_{13}, I_{14} ;
I_{23} I_{24} I_{34}

From configuration diagram

$$\frac{V_B}{I_{13} I_{34}} = \frac{V_A}{I_{23} I_{13}}$$

$$V_B = V_A \times \frac{I_{13} I_{34}}{I_{23} \times I_{34}}$$

In this way velocity of B can be found. As ω, $I_{12} I_{23}$ is given so, V_A is found & with result of V_A, V_B can be found easily.

Numerical (Theory of machine Page No. 138) Q.No. 1

1. Locate all the instantaneous centres for a four bar mechanism as shown. The lengths of various links are : AD = 125mm ; AB = 62.5 ; BC = CD = 75mm. If the link AB rotates at a uniform speed of 10 r.p.m in the clockwise direction, find the angular velocity of the links BC & CD.

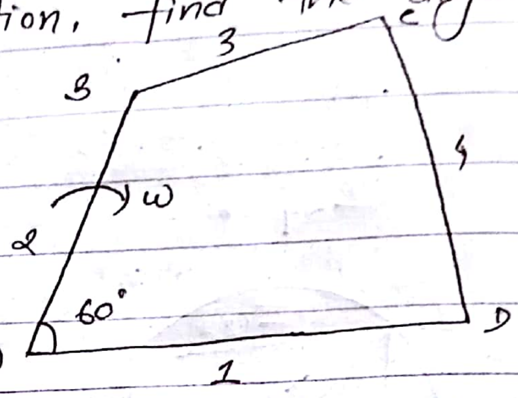


Fig:- Rough sketch

Soln:

No. of IC = $\frac{n(n-1)}{2}$

Here, n = 4 so, no = $\frac{4(4-1)}{2}$

$= \frac{4\times3}{2}$

$= 6$

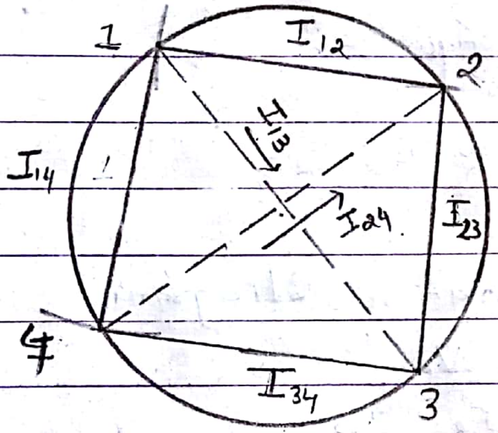


Fig: Circle Diagram

Scale:-

$125mm = \frac{125}{25}$ cm

$= 5$ cm

i.e. $1mm = \frac{1}{25}$ cm

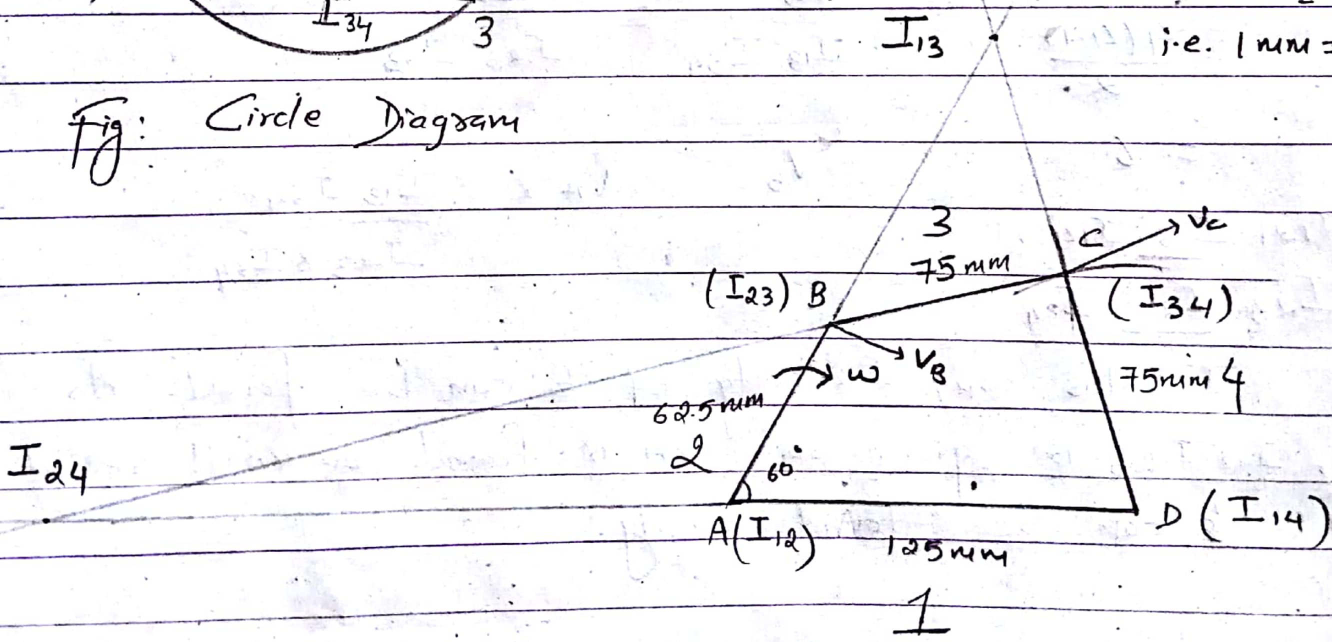


Fig. configuration diagram

Now, Calculation:-

$\omega_{AB} = ?$

$N_{AB} = 10$ r.p.m

$\therefore \omega_{AB} = \frac{2\pi N}{60}$

$= \frac{2 \times \pi \times 10}{60}$

$= 1.047$ rad/s

$\vec{V_B} = \omega_{AB} \times AB$

$= 1.047 \times (62.5) \times 10^{-3}$

$= 0.065$ m/s

From conf. digg:-

$\omega_{AB} = \omega_{A I_{13}}$ & $\omega_{BC} = \omega_{CB}$.

Now,

$\vec{V_B} = \omega_{BC} \times I_{23} I_{13}$

$0.065 = \omega_{BC} \times I_{23} I_{13}$ —(1)

measuring we get

$I_{23} I_{13} = 4.1$ cm and by scale factor

$I_{23} I_{13} = 4.1 \times 25$

$= 102.5$ mm $= 0.1025$ m.

From eqⁿ (1)

$\omega_{BC} = \frac{0.065}{0.1025}$

$= 0.63$ rad/s

By measuring, $I_{13} I_{34} = 2.9$ cm

$\therefore I_{13} I_{34} = 2.9 \times 25 = 72$ mm $= 0.072$ m

Process:

1) looking to the circle dia. (1,2,3) I_{12} & I_{23} coincide with I_{13} so, from A to B it meet I_{13}.

lly, I_{14} & I_{34} meet to I_{13} so, from D to C to I_{13} (In $\triangle$ 134)

2) And another point B also located as above.

To find ω_{CD}:

$\omega_{BC} = \omega_{CB}$

$\frac{V_B}{I_{23} I_{13}} = \frac{V_C}{I_{34} I_{13}}$

$\frac{0.065}{0.1025} = \frac{V_C}{0.075}$

$\therefore V_C = 0.0475$ m/s

Now,

$V_C = \omega_{CD} \times I_{13} I_{34}$

$0.0475 = \omega_{CD} \times 0.072$

$\therefore \omega_{CD} = 0.65$ rad/s

Ans:

$\therefore \omega_{BC} = 0.63$ rad/s (↺)

$\omega_{CD} = 0.65$ rad/s (↻)

Slider Crank Mechanism:-

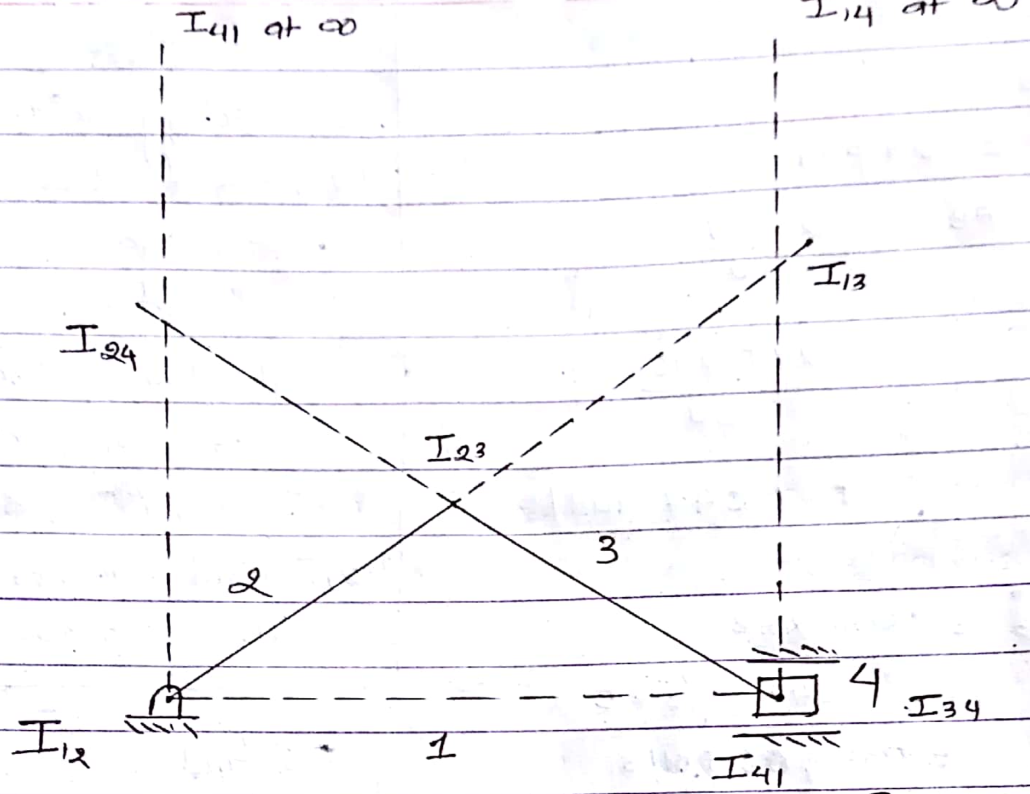


Fig:- Configuration Dia

$$\text{No. of IC} = \frac{n(n-1)}{2}$$

$$= \frac{4(4-1)}{2}$$

$$= 6$$

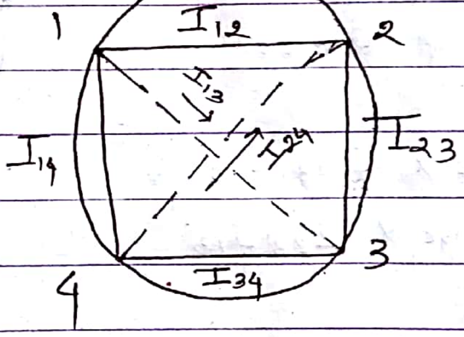
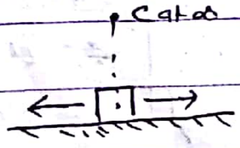


Fig: circle diag.

Note:-

Slider moves in horizontal direction only ie. I_{14} at ∞



So, its centre is at infinite (any point on line have its centre at infinite)

Numericals:- (Theory of machine page No. 128)

8. Locate all the instantaneous centres of the slider crank mechanism as shown in fig. 6.12. The lengths of crank OB and connecting rod AB are 100mm and 400mm respectively. If the crank rotates clockwise with an angular vel. of 10 rad/s. ($\measuredangle 45°$)

find 1) Vel. of slider A

2. Angular vel. of connecting rod AB.

location of instantaneous centre:

Since there are 4 links so, No. of IC (N) = $\frac{n(n-1)}{2}$

$= \frac{4(4-1)}{2} = 2 \times 3$

$= 6$ //

Scale

100mm = 2cm

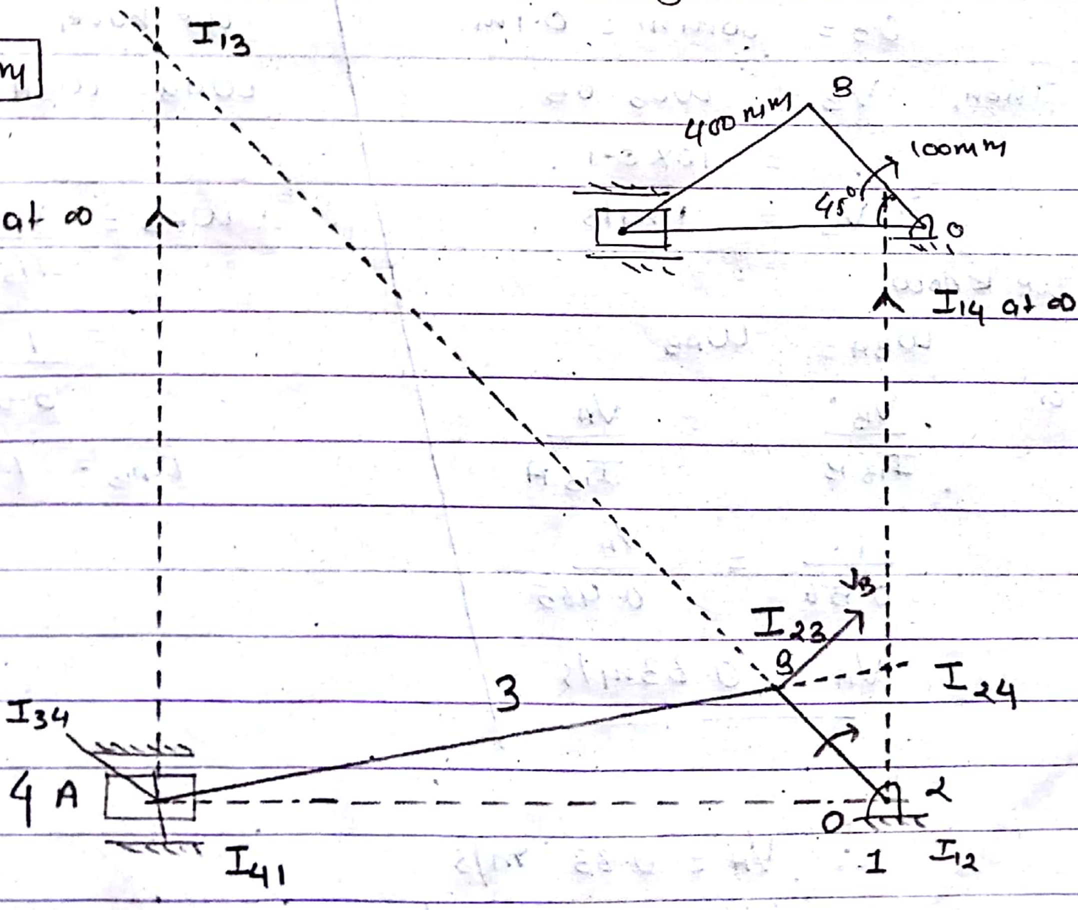
$\therefore$ $\boxed{1cm = 50mm}$

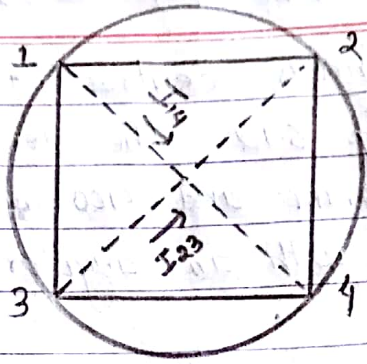
OR

$\boxed{1mm = \frac{1}{50} cm}$

I_{14} at ∞

Rough sketch





We have:

By measurement ; $I_{13}A$ = 9.3cm $\therefore I_{13}A = 9.3 \times 50 = 465$ mm

$I_{13}B$ = 11.2cm $I_{13}B = 11.2 \times 50 = 560$ mm

Q. 1 (soln);-

Now,

$\omega_{OB} = 10 rad/s$

$OB = 100mm = 0.1m$

Then, $V_B = \omega_{OB} \cdot OB$

$= 10 \times 0.1$

$\therefore V_B = 1 m/s$

We know,

$\omega_{BA} = \omega_{AB}$

or, $\frac{V_B}{I_{13}B} = \frac{V_A}{I_{13}A}$

$\frac{1}{0.56} = \frac{V_A}{0.465}$

$V_A = 0.83 m/s$

Q.No.2 (soln):-

$\omega_{AB} = ?$

We have,

$\omega_{AB} = \omega_{BA} = \frac{V_B}{I_{13}B} = \frac{V_A}{I_{13}A}$

$\therefore \omega_{AB} = \frac{V_A}{I_{13}A}$

$= \frac{1}{0.56}$

$\omega_{AB} = 1.78 rad/s$

$\therefore V_A = 0.83$ m/s

ω_{AB} 1.78 rad/s